A large Markovian linear program to optimize dairy herd net income. By V.E. Cabrera, pg. 000. A novel bio-economic optimization model for dairy herd dynamics was developed to study the impact of replacement policies that optimize dairy herd net income and nitrogen excretion. The model was used to study 5 diets. High concentrate diets maximized net revenue and high forage diets minimized N excretion. The optimal cow replacement policy varied for each scenario, but in general, the model always suggested to: (1) keep pregnant cows, (2) keep primiparous cows longer than multiparous cows, and (3) decrease replacement rates when income over feed cost is high. Implementation of the model is an important advancement in dairy decision-making.

OUR INDUSTRY TODAY

A large Markovian linear program for replacement policies to optimize dairy herd net income for diets and nitrogen excretion

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ABSTRACT

The purpose of the study was twofold: 1) to propose a novel modeling framework using Markovian linear programming to optimize dairy farmer defined goals under different decision schemes and 2) to illustrate the model with a practical application testing diets for entire lactations. A dairy herd population was represented by cow state variables defined by parity (1 to 15), month in lactation (1 to 24), and pregnancy status (0 non-pregnant and 1 to 9 month of pregnancy). A database of 326,000 lactations of Holsteins from AgSource DHI service was used to parameterize reproduction, mortality and involuntary culling. The problem was set up as a Markovian linear program model containing 5,580 decision variables and 8,731 constraints. The model optimized the net revenue of the steady state dairy herd population having two options in each state: keeping or replacing an animal. Five diets were studied to assess economic, environmental, and herd structural outcomes. Diets varied in proportions of alfalfa silage (38 to 98% DM), high moisture ear corn (0 to 42% DM), and soybean meal (0 to 18% DM) within and between lactations, which determined dry matter intake, milk production, and N excretion. Diet ingredient compositions ranged from one of high concentrates to alfalfa silage only. Hence, the model identified the maximum net revenue that included the value of nutrient excretion and the cost of manure disposal associated with the optimal policy. Outcomes related to optimal solutions included the herd population structure, the replacement policy and the amount of N excreted under each diet experiment. The problem was solved using the Risk Solver Platform® with the Standard LP/Quadratic engine. Consistent replacement policies were to: (1) keep pregnant cows, (2) keep primiparous cows longer than multiparous, and (3) decrease replacement rates when milk and feed prices are favorable. The optimal policy called for the replacement of open cows between 7 and 12 mo of lactation depending on parity, diet and market conditions. Under favorable market conditions, net revenue was highest with the highest concentrate diet, which was \$15.24 and \$52.32/mo per cow higher than the optimal net revenue realized with the intermediate and the no concentrate (all-forage) diets, respectively. A sub-optimal solution to limit the N excretion to 12 kg/mo per cow when market conditions were favorable resulted in a diet with the second highest level of concentrates to become the one with the highest net revenue. Under unfavorable market conditions, the diet with the highest concentrate content had the lowest net revenue than all the others. A sub-optimal solution of a maximum N excretion of 12 kg/mo per cow with unfavorable market conditions resulted in the least concentrate diet to have the highest net revenue (\$22/mo per cow) followed by the second highest concentrate diet (\$20/mo per cow) and the all-forage diet (\$18/mo per cow). The implementation of a Markovian linear program for dairy decision-making provides both robustness and versatility in operations research. The model could become a valuable tool for dairy farms to support economic decision-making.

Key Words: Replacement policy, system analysis, diet optimization, dynamic programming

INTRODUCTION

Economic optimization of dairy herd performance encompasses a series of research questions that still require substantial investigation. For instance, the voluntary cow replacement problem and the optimal number of reproduction services continue to be answered inadequately. Although voluntary culling decisions are critical, they are usually based on intuition and not on a systematic economic analysis because of the lack of metrics and methods that could certify that decisions are right.

Since the 1980s dynamic programming (**DP**) based on Bellman's principle of optimality (Bellman, 1957) has been the most recognized method to optimize dairy herd economics. Several studies have used DP for dairy economic decision-making advancing the knowledge of dairy herd optimization. For example Van Arendonk (1984; 1985; 1986; 1988) and Van Arendonk and Dijkhuizen (1985) studied the replacement and reproduction policy in dairy cattle. Kristensen (1987) introduced the concept of hierarchic Markovian processes and Kristensen (1988, 1991) reformulated the concept of policy and value iteration with substantial gains in computational efficiency to solve DP problems. Later Kristensen and Thysen (1991) developed a model that accounted for milk quotas in Europe, and in the US DeLorenzo et al. (1993) developed the largest DP model at the time with 151,200 cow states to solve the replacement problem. Also Kristensen (1993) combined Bayesian probabilities with Markovian decision processes in a DP model to calculate optimal replacement rates and Stott (1994) found by DP that keeping milking cows longer in the herd would increase the net profit of British dairy farms. More recently, De Vries (2004; 2006) has been applying DP and Markovian simulation processes together to solve and simulate the replacement and reproductive problems, offering interesting advancements in the decision process. De Vries (2004) provided a new algorithm to account for seasonally delayed replacement of cows and De Vries (2006) provided new algorithms to calculate the value of a pregnancy.

However, a potential problem of traditional DP formulation is that the model can easily become very large and complicated with limited applicability to real-life problems (Smith et al., 1993). An additional challenge with the traditional DP formulation is the insufficient number of parameters selected by the decision-maker, which limits its applicability (Groenendaal et al., 2004). Lehenbauer and Oltjen (1998) stated that more effort has been devoted to constructing models than on applying these models in real-life farm decision-making. Because of these reasons, Groenendaal and colleagues (2004) found it worthwhile to revisit the marginal net revenue technique with the justification that this simpler method easily could be applied in final users' decision-making through decision support systems without compromising the accuracy of the outcomes. An appealing method that simplifies the search for the optimal policy was described by Hillier and Lieberman (1986). The method is the solution of the original Bellman equation by using linear programming (**LP**).

A formulation of the DP problem as a LP algorithm would allow the inclusion of interaction between herd mates in a particular herd. Yates and Rehman (1998), the only study of its class, used a LP formulation of the Markovian decision process for the dairy replacement problem accounting for the performance of the whole herd in addition to the performance of each individual animal. The Yates and Rehman (1998) model was rather small having only parity as the state variable (1 to 12) and consequently their results could be used as good conjectures, but not as conclusive findings.

Linear programming formulation of DP problems overcomes another limitation of the traditional DP method. The LP formulation allows the possibility of solving a problem for user-defined sub-optimal conditions. The sub-optimal condition of a DP formulation has been referred to as an unsolvable limitation of traditional DP formulation (Dekkers, 1991), and has remained unsolved hitherto in the literature. Another advantage of LP formulation over traditional DP formulation is that once the problem has been defined, standard LP optimization algorithms can solve the problem efficiently to explore a number of different research questions. The formulation of a Markovian LP problem allows the investigator and hopefully the end-users to better interact with the model for decision-making. The solution of a DP as a LP problem would also allow managing efficiently for different time steps in the dynamic processes.

The goal of this study is therefore to formulate a large Markovian LP model capable to solve a real-farm dairy herd replacement DP problem including sub-optimal solutions.

MATERIALS AND METHODS

Production Scenario

A typical adult Holstein herd was represented and optimized. In the model, a cow started her productive life after first calving. Each cow produced milk for a variable length of time depending on her reproductive status, probability of survival, and probability of involuntary culling. A pregnant cow (**PREG**) was dried-off two months before expected calving. At calving, the cow was assigned to the next parity (**PAR**) category and started a new productive cycle. When a cow had 2 mo of lactation (**MIL**) a reproductive program was started and continued through successive pregnancy service attempts. The pregnancy rate determined the probability of a cow becoming pregnant in a month. If conception failed, cows continued to be serviced in subsequent estrus cycles. This protocol repeated for each parity under this structure. The model selected the optimal time and cow state of voluntary culling that maximized net revenue and found the herd steady state. By implication the model also decided the optimal number of reproductive services that should be performed.

The adult population of cows in a herd could be realistically described as successive steps through change of states established by transition probabilities (Cabrera et al., 2006a; 2008). The time step in the model was one month, consequently all production (e.g., milk), economic (e.g., milk price), and biological parameters (e.g., pregnancy rate) were defined on a monthly basis. Therefore, every month, every cow in the herd was run through a probabilistic process that included a number of potential options. For example the following processes were defined by transition probabilities, which were repeated for parities 1 to 15, Fig. 1. In every state and stage there was always a possibility for a cow to be involuntarily culled or unexpectedly die. When a cow survived from one state to the next, the cow became one MIL greater (except when the cow calved and became one parity older), Fig. 1. When a cow was eligible for breeding, the cow could become pregnant and when the cow was pregnant, the cow could abort.

In addition to the transition probabilities and potential states exemplified in Fig. 1, cows could be voluntarily culled in any state at any point in time. Voluntary culling was handled by the model as the decision variable to maximize the net revenue.

As seen, the dimensions of the model were large enough to accommodate all potential cow states. These could be represented by vector matrices: PAR = 1 to 15, MIL = 1 to 24, and PREG = 0 (non-pregnant) and 1 to 9 (monthly pregnancy states). Therefore there were 15 x 24 x 10 = 3,600 possible states. However, to comply with the reproductive program that starts in MIL = 2, the MIL has to be at least 2 units larger than the pregnancy status (MIL > pregnancy state + 2), 54 possible states were excluded from each parity and 810 states for the 15 parity combinations. Consequently, the effective number of cow states was 186 per parity and 2,790 for all 15 parities.

The model started with one cow in PAR = 1, MIL = 1 and PREG = 0. Then, the stochastic Markovian processes, using transition probabilities, distributed the proportions of such cow to all potential states in the model. Through recursive iterations, this distribution across states reached a steady state. This iterative process was solved by standard LP algorithms.

Markovian Decision Processes and Linear Programming

The population dynamics in a dairy herd were described as Markovian processes. Markovian processes are a special case of stochastic processes, in which it is possible to analytically track the stochastic processes through the Markovian property that indicates the conditional probability of any future event given any past event (Hillier and Lieberman, 1986). A Markovian decision model is then used to find a decision policy to optimize net revenues.

The model was then solved by linear programming as a modified Markovian dynamic program problem described as follows. The objective function maximized the net revenue of the vector decision made in each state. The matrix had 5,580 terms (2 decisions in each state by 2,790 states). Consequently, the solution vector had 5,580 terms. Therefore the:

$$\text{Optimum economic solution} = \max \sum_{i=1}^{2790} \sum_{k=1}^{2} y_{ik} NR_{ik} \qquad [1]$$

where i is the state and k is the decision to be made (1 = keep and 2 = replace). Consequently y_{ik} is the steady state proportion of state i when decision k is made and NR_{ik} is the net revenue expected for the state i when decision k is made. For example y could be 0.003 and NR could be \$63.46 for $i = 222$ and $k = 1$. This means that the proportion of cows in PAR = 2, MIL = 9 and PREG = 6 (i = 222) would be 0.003 when the herd reaches steady state. This proportion of cow (group of cows) would produce a monthly net revenue of \$63.46. Equation 1 found the maximum *NR* of the optimum replacement policy. The optimum replacement policy indicated the optimal time of replacing a cow to maximize the *NR* depending on the states of MIL, PREG and PAR.

The model had 8,731 constraints:

The constraints of non-negativity of all decision variables,

$$y_{ik} \geq 0 \text{ for all } i \text{ and } k \qquad [2]$$

the constraint that assured that herd size remains constant, so the sum of proportions at steady state were equal to 1,

$$\sum_{i=1}^{2790} \sum_{k=1}^{2} y_{ik} = 1 \qquad [3]$$

and 2,790 constraints (one for each state) that found the steady state probabilities,

$$\sum_{k=1}^{2} y_{ik} - \sum_{i=1}^{2790}\sum_{k=1}^{2} y_{ik} P_{ijk} = 0 \text{ for } j = 1 \text{ to } 2{,}790 \quad [4]$$

where P_{ijk} is the ij^{th} element of the transition matrix resulting from making decision *k*. The P_{ijk} were based on the transition probabilities obtained from commercial records described later. The model accounted for animals moving from one state to a successive potential state determined by the law of probabilities contained in the transition matrices of probabilities of pregnancy, mortality, involuntary culling and abortion rates defined by PAR, MIL, and PREG.

Net Revenue when Keeping the Cow

The NR_{ik} (net revenue for decision 1 = keep the cow) was calculated as a function of six economic factors: the milk income over feed cost (*IOFC*), the income of a new born calf (*INB*), the cost of a dead cow (*CDC*), the cost of involuntary culling (*CIC*), and the cost of insemination (AI). Additionally, a function of the cost of manure disposal and the value of nutrients excreted was included in the net revenue calculation (*EnvFactor*):

$$NR_{i1} = IOFC_i + INB_i - CDC_i - CIC_i - AI_i + EnvFactor_i \text{ for } i = 1 \text{ to } 2{,}790 \quad [5]$$

The milk income over feed cost (*IOFC*) was calculated as the milk value (*Mv*) less the feed cost (*Fc*). The milk value (*Mv*) was the multiplication of milk production (*MP*) by milk price (*Mp*). The feed cost (*Fc*) was calculated as the value of the DMI, which was a function of the diet, PAR, MIL, and PREG. The diet was defined as the proportion of feed ingredients of alfalfa silage (*F*), high moisture ear corn (*C*), and soybean meal (*SBM*) that are described later as part of the experimental design.

$$IOFC_{i1} = Mv_i - Fc_i = MP_i * Mp - DMI_i * (F\% * Fp + C\% * Cp + SBM\% * SBMp)$$

for i=1 to 2,790 [6]

The income of a new born (*INB*) was the value of a new born calf as defined by the economic value of a new born male or female and their respective probabilities. The probability of a female new born was set at 46.7% (Silva del Rio et al., 2007). The income of a new born (*INB*) was realized during the 9th month of pregnancy (PREG = 9).

$$INB_{i1} = 0.467 * FCp + (1 - 0.467) * MCp \text{ for } i = 1 \text{ to } 2{,}790 \text{ and PREG} = 9 \qquad [7]$$

where *FCp* is the price of a female calf and *MCp* is the price of a male calf.

The cost of disposal of a dead cow (*CDC*) was assessed as the composite cost of disposal and the cost of bringing a replacement. These costs were partially offset by the value of a new born:

$$CDC_{i1} = Mr_i * (Dc + HRc - INB_i) \text{ for } i = 1 \text{ to } 2{,}790 \qquad [8]$$

where *Mr* is the mortality rate, *Dc* is the disposal cost, and *HRc* is the heifer replacement cost.

The cost of involuntary culling (*CIC*) was assessed as the probability of involuntary culling (ICr) multiplied by the difference between heifer replacement cost and the value of a new born less the salvage value (Sv) realized when culling a cow.

$$CIC_{i1} = ICr * (HRc - INB_i - Sv) \text{ for } i = 1 \text{ to } 2{,}790 \qquad [9]$$

The cost of artificial insemination (*AI*) was calculated as the monthly estimated cost of a common reproductive program using artificial insemination including labor, semen, and pregnancy diagnosis and was estimated at $20/mo (AgFA Wisconsin 2008, http://cdp.wisc.edu/AgFA.htm), which was charged to open cows in reproductive status (MIL ≥ 2 and PREG = 0).

The model included an environmental function applied to the manure and nutrient balance of the dairy herd system. The nutrient excreted (*NutValue*) that could be used for crop production less the cost of manure disposal (*CMD*) was included in the model. The cost of manure disposal was defined as a function of loading, transporting, unloading and incorporating the excreted manure in nearby crop fields (Hadrich et al., 2008).

$$EnvFactor_{i1} = NutValue_i - CMD_i \text{ for } i = 1 \text{ to } 2{,}790 \qquad [10]$$

The cost of manure disposal was calculated to be $16.50/mo per cow and was charged equally to all cows assuming that the farm has a manure storage facility that can hold manure for at least six months and assuming that available fields to apply the manure are within a 8 km radius (Hadrich et al., 2008) and that the farm has cropland available to apply all produced manure. The nutrient excreted (*NutValue*) was calculated as a function of the N content of the manure. Nitrogen excreted was calculated as the difference of N ingested (DMI * CP/6.25) and the N exported in the milk as the milk protein (MP_i * MilkProtein/6.38; DePeters and Cant, 1992). Both N ingested and N excreted were defined as part of the experimental design of the studied diets (see later). The value of N was calculated as its value of fertilizer in crop fields assumed to be equivalent to the value of N content of urea: UreaValue * 46% N content. This value then was multiplied by a factor to account for the value of other nutrients available in the excreted manure such as P, K and microelements. The multiplication factor was set at 3, assuming that the full value of manure is 3 times the value of N as fertilizer (Pennington et al., 2009).

Net Revenue when Replacing the Cow

The net revenue for decision 2 (replace the cow) considered the voluntary replacement of a cow by a pregnant springer that enters the herd just before calving. She then calved and became cow in PAR = 1, MIL = 1 and PREG = 0. Immediate replacement to maintain the herd population is a standard assumption in DP (De Vries 2006). As previously stated, DP requires reaching a steady state of the herd population dynamics to reach an optimal solution and this is achieved when the population remains stable, which also reflects practical farm operation. The net revenue for replacing a cow was calculated as the difference of the salvage value of the cow leaving the herd less the difference of the heifer replacement cost (*HRc*) and the income of a new born (*INB*).

$$NR_{i2} = Sv - (HRc - INB) \text{ for } i\text{=1 to 2,790} \qquad [11]$$

Computer Model

The problem was set as a large Excel® spreadsheet (5,580 columns and 2,790 rows) and solved using the Risk Solver Platform with the Standard LP/Quadratic Engine.

Reproduction and abortion rates

Monthly pregnancy rates according to the dimensions of the model were obtained from a Midwest DHIA program (AgSource DHI Cooperative Services, see acknowledgment) that included 326,000 Holstein lactations during a 5-yr period (2003 to 2007). Records included the actual number of cows becoming pregnant from one month to the next. Consequently, the probability of pregnancy occurring during a particular MIL was calculated by dividing the number of cows becoming pregnant during a month by the number of eligible cows the previous month. These values then represented the product of conception rates by service rates in every month. Table 1 shows primiparous and multiparous cows' pregnancy rates that were used in the model.

Abortion rates were not available in the DHIA database. These were obtained from De Vries (2006), which indicated a probability of abortion by month of gestation (2 to 8) of 3.5, 2.5, 1.5, 0.5, 0.25, 0.1, and 0.1%, respectively, with a total probability of abortion of 8.45% during a pregnancy. The probability of abortion in the first month was set at 0% because fetal embryonic loss during the first month was considered as if the cows did not get pregnant.

Mortality and Involuntary Culling Rates

Monthly mortality rates (*Mr*) according to the dimensions of the model were obtained from the AgSource DHI Cooperative Services program that included 326,000 Holstein lactations during a 5-yr period (2003 to 2007). The database included records of number of cows that died in a particular month, which was used to calculate the mortality rate (*Mr*) by MIL and PAR. The database, however, did not report involuntary culling. Involuntary culling (*ICr*) was calculated as a function of mortality rate. Based on AgSource benchmark data for Holsteins (AgSource Cooperative Services, 2009), it was safe to assume the involuntary culling to be 3 times higher than the mortality rate (Fig. 2). As expected, mortality and involuntary culling are higher in early lactation (right after calving), decreasing towards mid-lactation and increasing again towards the end of lactation. Multiparous cows have higher mortality and involuntary culling than primiparous cows.

Economic Factors

Average market conditions observed for Wisconsin in 2008 were used as baseline. Milk price (*Mp*) was set at $0.44/kg ($18.92 per 100 lb) (Understanding Dairy Markets website, accessed 10 March 2009, http://future.aae.wisc.edu), the heifer replacement cost (*HRc*) at $2,000 (Wisconsin USDA Agricultural Marketing Service Reports, 2008), the meat value at $1.16 per kg (Understanding Dairy Markets website, accessed 10 March 2009, http://future.aae.wisc.edu) and the salvage value (*Sv*) of a 726-kg culled Holstein cow at $840.32. The disposal cost of a dead cow (*Dc*) including labor and machinery was estimated at $100. The price of a female calf (*FCp*) was set at $500 and the price of a male calf (*MCp*) at $50 (Wisconsin USDA Agricultural Marketing Service Reports, 2008). The costs for feed ingredients was set at ($ per kg) 0.115 for alfalfa silage (calculated from alfalfa hay), 0.187 for high moisture ear corn, and 0.366 for soybean meal (Understanding Dairy Markets website, accessed 10 March 2009, http://future.aae.wisc.edu). The value of urea was set at $0.6071/kg (USDA Economic Research Service, 2008)

Experimental Design

Five diet treatments were studied. The model was solved for each one of the diets under different price scenarios to study the sensitivity of the outcomes to market conditions. The measured outcomes included the herd population structure, the replacement policy, the net revenue, and the amount of N excreted under each diet treatment. A sub-optimal policy was also studied that included an imposed maximum N excretion of 12 kg/cow per mo. A level of maximum N excretion of 12 kg/cow per mo was empirically found by testing the model outputs under different scenarios.

Dietary Treatments

Milk production, milk protein, and DMI for entire Holstein lactations in response to diets defined in proportions of alfalfa silage, high moisture ear corn and soybean meal to match every state defined in the model were based on a large controlled study in Wisconsin (Tessmann et al., 1991). No other study that could accommodate the objectives of this research was found in the literature.

Data from Tessmann et al. (1991) reported in weeks of lactation were aggregated at the monthly level to be integrated in the model created in this study. Following Tessmann et al. (1991) lactation was divided into three categories: early (1-3 mo), mid (4-7 mo) and late (8-22 mo). The proportion of alfalfa silage, high moisture ear corn, and soybean meal varied for each diet in each lactation stage, as shown in Table 2. As in Tessmann et al. (1991) diets were iso-nitrogenous balanced at 19% CP in early and 17% CP in mid and late lactation and dry cows received 11.4 (primiparous) and 15.9 (multiparous) kg DMI/d with 17% CP. For each formulation, 2% of vitamins and mineral supplements were assumed to complete the diets, which were equal for all diets and were not included in the cost function.

Tessmann et al. (1991) reported results only to 44 weeks in lactation as it is usual to have a large proportion of the cows finishing their lactation around 11 mo after calving. However, the model structure required data beyond these months. To complete these data with the model structure, individual trend lines were fitted for weeks 45 and later. A persistence factor was individually calculated for each lactation curve between the week in which a peak was reached and week 44. The persistence factor was used to complete curves until MIL = 24. The DMI and milk protein were similarly adjusted after 44 weeks in lactation to represent a similar function and trend as the milk production. The implications of this assumption are not critical in the model results because more than 90% of the herd population is contained within MIL ≤ 11. Therefore, only less than 10% of the herd population is affected by the extrapolation. Furthermore, an additional 8% of the herd population is between MIL 12 and 15 where the extrapolated points are near the last points of the original data.

Primiparous cows had flatter and more persistent milk lactation curves whereas multiparous cows had less persistent lactation curves with higher peaks. Lactation peaks occurred between MIL 2 and 3, after which milk production decreased towards the end of lactation. The DMI followed the trend of milk production, so increased in early lactation, and decreased in late lactation. Milk protein that decreased in early lactation and then increased steadily towards the end of lactation was similar between primiparous and multiparous cows, although it had a higher response to diet composition in primiparous cows. The all-forage diet (diet 5) that had 98% alfalfa silage composition throughout the lactation showed substantially lower milk production in early and mid lactation than the other diets for both primiparous and multiparous cows. The highest concentrate diet (diet 1) showed overall higher milk production in early lactation, although it was very comparable with the second highest concentrate diet (diet 2) for multiparous cows in mid and late lactation. For more details about the original study, please refer to Tessmann et al. (1991).

RESULTS AND DISCUSSION

Optimal Net Income Policy

The optimal replacement policy with favorable market conditions consistently called for replacing cows that were open at a certain month in lactation, depending on market conditions, parity and diet. With only small variations, the replacement policy was similar for diets containing concentrate (diets 1 to 4). The optimal policy for 2008 market conditions and diets 1 to 4 called to replace open primiparous cows on MIL 11 and multiparous open cows on MIL 10. With the all-forage diet (diet 5) the optimal policy called to replace open primiparous cows on MIL 12 and multiparous open cows on MIL 11 (Table 3). When there was an unfavorable market defined with a low milk price ($0.22/kg), high corn price ($0.24/kg) and low replacement cost ($1,500), the policy for diets 1 to 4 called for replacing primiparous open cows on MIL 9 and for multiparous open cows on MIL 8. For the diet 5 the replacement policy called for replacing open cows on MIL 10 whether primiparous or multiparous. Along with the replacement policy, the model selected the maximum number of reproductive services to optimize farm net revenue. The last month in which reproductive services should be attempted was a month before the replacement month. For example, if the policy called to replace open cows on MIL 11, then reproductive services should be provided only until MIL 10.

In agreement with other studies (e.g., Groenendaal, 2004; De Vries, 2004; 2006), farmers' practices and logical reasoning, it is not an economical decision to voluntarily replace pregnant cows. The future net revenue realized from a pregnant cow (the reward of new born and future lactations milk production) is always higher than the potential benefit realized by a potential replacement. However, if a cow reaches a certain MIL without becoming pregnant, the future net revenue realized of her replacement will exceed the potential net revenue derived from a pregnancy at that point in time or later. The optimal replacement time measured as the MIL of voluntary culling is a critical decision that can be found with the model. Whereas replacement due to mortality and involuntary culling occurs with little or no action of the manager, the replacement from voluntary culling requires a thorough evaluation comparing the actual cow with a potential replacement in the long-term, which includes several lactations in the future and potential replacements.

The modeling results indicated that primiparous cows should be given more chances than multiparous cows of getting pregnant. This result is in agreement with previous reports (Dekkers et al., 1998; Groenendaal et al., 2004; De Vries, 2006). Older cows are recommended for one less breeding and are culled one month earlier in lactation because their milk production is declining at a faster rate than the rate of primiparous cows. This may also be because older cows are at greater risk of involuntary culling.

Diet 5 held cows longer than diets 1 to 4. One reason for this result was that cows consuming the all-forage diet, although less productive, had markedly more persistent lactation curves than the other diets, which consequently realized higher monthly net revenues when keeping cows longer. Diets containing concentrates (diets 1 to 4) had similarly shaped lactation curves among them although the diet with the highest concentrate (diet 1) had the highest net revenue of all diets.

It can be speculated that intensive feeding systems (i.e., using high concentrate levels in the diets, diets 1 to 4) could lead to a greater net revenue earlier in the lactation because of a high milk response to concentrates (Earleywine, 2001), which consequently would justify earlier replacement policies to have more cows close to the peak of lactation. In contrast, under an all-forage diet (diet 5), not that much pressure would exist for intense replacement policies earlier in lactation because of a lower response of milk production to lactation stage and a greater persistency. Results of the model showed that keeping a cow longer with diet 5 would bring more benefits than replacing a cow.

Favorable market conditions like those experienced in 2008 (high milk price, intermediate corn cost and high replacement cost), called for lower replacement rates than unfavorable market conditions (low milk price, high corn price and low replacement cost) (Table 3). Under these defined unfavorable market conditions, the model tries to allocate most of the cows to the peak of the lactation curve (where the milk income over feed cost increases) by increasing the replacement rate, which is supported by the fact that the replacement cost is relatively lower. In the unlikely situation that the unfavorable market conditions of low milk price and high corn price are combined with a high replacement cost (e.g., $2,000), the model would suggest to keep cows longer (up to 15 MIL, data not shown) than with the defined 2008 favorable market conditions.

Herd Structure

Table 4 is a steady state Markovian representation of second parity cows for 2008 market conditions fed diet 1 that maximizes dairy herd net revenue. The model finds the steady state of the herd as the equilibrium of cows entering the herd and cows leaving the herd taking into account the transition probabilities defined as the probabilities of a cow becoming pregnant, being involuntarily culled, dying, or voluntarily culled. Voluntary culling was a decision of the optimization model. The sum of all the coefficients in Table 4 (0.231) represents the proportion of cows of the whole herd standing in PAR = 2. Table 4 indicates that the proportion of cows starting second parity is 0.024, which is equivalent to saying that the category PAR = 2, MIL = 1 and

PREG = 0 will have in equilibrium 2.4% of the herd (Fig. 3). The proportion of cows finishing PAR = 2 is 0.012 (1.2% of the herd), which is the sum of the coefficients in the last column in Table 4. Consequently, the difference between these two numbers or (0.024 – 0.012), 0.012 indicates the proportion of cows that were voluntarily or involuntarily culled during PAR = 2. The proportion of voluntarily culled cows in PAR = 2 (decided by the model) was 0.0084 cows, which is found as the difference between open cows (PREG = 0) in MIL = 9 and the proportion of cows becoming pregnant (PREG = 1) in MIL = 10 (0.008862 - 0.000465). For each solution there were 15 tables similar to Table 4, one per parity as part of the solution of the model.

The optimal proportion of the herd population for diets 1 to 4 was similar among these diets: 0.496, 0.243, 0.117, 0.057, and 0.053 for parity 1, 2, 3, 4, and 5 to 15, respectively (Fig. 3a). As expected, the majority (85.7%) of the population is contained in the first 3 parities and only a small fraction of animals reaches later parities. Only 2.8% of animals would be in parity 5, and the proportion of cows reaching parity 10 or higher could be considered negligible. For the all-forage diet (diet 5), the optimal structure was: 0.482, 0.244, 0.122, 0.061, and 0.061 for parity 1, 2, 3, 4, and 5 to 15, respectively (data not shown). Again, the majority (84.8%) of the population is contained in the first 3 parities. Only 3.1% of animals would be in parity 5, and less than 0.2% of them will reach parity 10.

As expected, the proportion of the herd population in increasing MIL categories decreases because of mortality, involuntary and voluntary culling (Fig. 3b). Similar results as those illustrated in Fig. 3b were found for the other diets containing concentrates (diets 2, 3 and 4 (data not shown)). About 9.1% of the herd population is in first MIL whereas only 0.1% is in MIL = 19. No cows reach MIL $\geq$ 20. The decrease in proportion of animals followed a linear trend for MIL = 1 to 10; but fell sharply for MIL > 10 because the optimal voluntary replacement policy calls to replace open cows in MIL = 11 (PAR = 1) and in MIL = 10 (PAR ≥ 2). For diet 5, the sudden decline starts one month later.

It is important to indicate that the above results show that the model was large enough to accommodate all potential cow states. Dimensioning the model for only 12 parities would probably be enough to accommodate a typical Holstein herd population. Knowing that the model only selects to replace open cows, the dimensions of the model could substantially be reduced by giving the probability of replacement only to open states.

Net Revenue

The net revenue profile varied with several factors as stated in Equations 5 and 11. The single most important factor influencing net revenue was the milk income over feed cost (Equation 6), which was heavily impacted by the diet ingredient composition and the milk price. For 2008 market conditions the net revenue per mo for diet 1 ranged between \$361 (PAR = 1, MIL = 2, PREG = 0) and -\$152 (PAR ≥ 2, MIL = 23, PREG = 0), whereas for diet 5 ranged between \$332 (PAR = 1, MIL = 3, PREG = 1) and -\$152 (PAR ≥ 2, MIL = 23, PREG = 0).

The calculated net revenues followed a pattern similar to the lactation curves: first, it increased and reached a peak and then decreased to the end of lactation. In early to mid lactation the highest net revenues occurred when a cow was found pregnant or when the cow was in early pregnancy. Later in lactation the highest net revenues occurred at parturition when the revenue of a new born was realized. The lowest net revenues in early and mid lactation occurred when the cows were open and in late lactation and in PREG = 8, the time when a cow is dry and not producing any milk revenues. The herd net revenues were the aggregation of all net revenues of all cows in a herd (proportion of cows by states) in a period of time of 1 mo. With exception of diet 5, the concentrates of other diets varied in proportion of high moisture ear corn, soybean meal and alfalfa silage throughout the lactation as seen in Table 2. With diets 1 to 4, the herd was simultaneously fed 3 different diets and consequently the net revenue and N excretion was the aggregation of these 3 diets weighted by the proportion of cows in these corresponding states.

Maximum Net Revenue

The optimization of the model under the baseline scenario with 2008 market conditions (favorable) found that diet 1 had the maximum net revenue of $132/mo per cow (Table 3). Diet 2 was only $0.27/mo per cow lower. The other diets fell substantially lower: Diet 3 (-$15.24), diet 4 (-$26.67) and diet 5 (-$52.32). As a reference, De Vries (2004) reported net revenues varying between $59 and -$39/mo per cow. Although concentrate prices in 2008 were intermediate (not the cheapest nor the most expensive), diet 1 that had the higher contents of high moisture ear corn and soybean meal was the economic optimum due in part to the high response of milk to concentrates together with the high price of milk during 2008. Note that these comparisons are performed at optimal management conditions for each diet scenario.

Diet 3 (the third concentrate diet) behaved differently than the other diets as it did not always follow expected patterns between diets 2 and 4 with regards to net revenue and N excretion (Table 3). Although it is difficult to know exactly what and how factors influenced these outcomes under the optimization framework, it is likely that the interaction among expected milk production, DMI and milk protein had the most influence on these results. On one side, the ratio of milk production over DMI determines dynamically the marginal income over feed cost and on the other side the relationship between milk production and milk protein determines dynamically the N excreted. Reviewing the original publication, Tesmann et al. (1991) reported no significant differences between diets 2, 3, and 4 for milk production and DMI. Interestingly, diet 3 had however a numerical higher DMI than diet 2 for primiparous cows. Examining the original data of

DMI together with milk production, it was found significantly lower feed efficiency (milk/DMI) of diet 3 compared to diet 4 ($\alpha < 0.001$) for both primiparous and multiparous cows. Consequently, under favorable market conditions that gives a higher weight to milk value and less to feed value, the net revenue of diet 3 followed an expected pattern being between diets 2 and 4. However, under unfavorable market conditions where the cost of grain had a relatively higher weight, the net revenue of diet 3 that had a relatively higher DMI per unit of milk produced and a lower net revenue than diet 4 when it would have been expected the opposite.

With unfavorable market conditions that included a low milk price ($0.22/kg), high corn price ($0.24/kg) and low replacement cost ($1,500) net revenues for all diets decreased substantially. With these unfavorable price combinations, diet 4 that had a low level of concentrate and high forage content (Table 2) would have the maximum net revenue of $22/mo per cow, followed by diet 2 ($21), and then diet 3 ($19). It is noteworthy that under unfavorable market conditions, diet 5 would have a higher net revenue than diet 1 ($18 vs. $15). Diet 1 under these market conditions would have the lowest net revenue of all. As seen, the model can help to select the maximum net revenue diet according to market conditions. These results are consistent with previous analyses (Østergaard et al., 1996; Tedeschi et al., 2000; Earleywine, 2001) that have found that diet manipulation could have an important impact on farm net revenue according to lactations and market conditions.

Nitrogen Excretion

Substantial differences were found among scenarios regarding N excretion. In general, higher concentrate diets were associated with higher N excretion (Mulligan et al., 2004). The model calculated the N excreted as the difference of N ingested and N exported with milk by the implied N efficiency utilization in milk production defined by the diets. Consequently, a lower N excretion is expected with diets with higher conversion rate from fed N to milk protein. Although a small amount of ingested N is biologically used for cow body maintenance and fetus nutrition, these were ignored in the model. Therefore, estimates of N excreted may have been overestimated. However, these overestimates should only be minimal due to the fact that the N used for body maintenance and fetus nutrition is only a very small proportion of the N ingested and commonly not included in similar analyses (e.g., Powell et al., 2008). Additionally, N excretion was equally assessed with all the scenarios and all diets so only minimal distortion would be expected because of this assumption among scenarios and diets .

Under optimal policies with market conditions for 2008, the lowest N excretion was found with diet 5 and was 11.35 kg N/mo per cow (Table 3) and the maximum N excretion was found with diet 1 and was 12.56 kg N/mo, a difference of 1.21 kg N/mo or 14.52 kg N/yr excreted per cow. The estimated N excretion for other diets was (kg N/mo per cow): diet 2 (12.47), diet 3 (12.55), and diet 4 (12.09). Although the N excreted was inversely associated with the level of concentrate in the diet and consequently with the level of DMI and with economic outcome (for 2008 market conditions), diet 3 was an exception. Diet 3 had higher N excreted than diet 2 and a level of N excreted very close to diet 1.

As with net revenues, diet 3 seemed to behave outside the patterns of contiguous diets. As previously discussed, the interaction of milk production and milk protein by diet 3 probably had the greater influence on these results. The original report from Tessmann et al. (1991) indicated that there were no significant differences among diets 2, 3 and 4 with respect to milk protein and the milk protein content of diet 3 was in line in between 2 and 4. However an analysis of the original data indicated that the difference between N ingested (calculated as a function of DMI and CP) and the N exported (calculated as a function of milk protein content and milk produced) was significantly higher for diet 3 than for diet 2 ($\alpha < 0.05$) for both primiparous and multiparous cows. Consequently, calculated N excreted from diet 3 was always higher than for diet 2 when it would have been expected to be lower. Under sub-optimal conditions with a constraint limiting the amount of N excreted, diet 3 needed a greater population adjustment than diet 2 to converge to a herd structure to comply with such restriction increasing the selection of cow states that yielded less N excretion rather than those states with higher net revenues.

The model could be used to help producers reach the maximum net revenue within the constraint of respecting a maximum limit of N excretion to the environment based on herd structure and diet ingredient composition (Cabrera et al., 2006b). Depending on the number of cows on the farm, the cropland and the environmental restrictions, fine-tuning of diets and herd structure to reach a goal of maximum N excretion could be critical. For example if the nutrient management plan of the farm indicates that N excretion per cow should be not more than 12 kg/mo per cow, the model could be solved accordingly for this sub-optimal condition.

Imposing a level of 12 kg N/mo per cow as a maximum N excreted under the 2008 favorable market conditions (high milk price, intermediate corn price and high replacement cost), the diets containing higher levels of concentrate complied with this restriction by drastically elevating the replacement rate (e.g., replacement suggested at 9 MIL instead of 11 MIL for diets 1 to 3), which evidently impacted the net revenue (Table 3). Therefore, it would cost \$12/mo per cow to reduce 0.56 kg of N excreted with diet 1, \$5.43/mo per cow to reduce 0.47 kg N excreted with diet 2, \$12.06/mo per cow to reduce 0.55 kg N excreted with diet 3 and \$0.55/mo per cow to reduce 0.09 kg of N excreted with diet 4. Under the N excretion constraint, diet 2 yielded the highest net revenue of \$126 (Table 3). For diet 5 the maximum N excretion of 12 kg/mo per cow was irrelevant and did not alter the optimal replacement policy because the optimal net revenue was found at a level that was lower than the limit imposed in N excretion. Imposing a level of 12 kg N/mo as a maximum N excreted under unfavorable market conditions (low milk price, high corn price and low replacement cost), diets 4 and 5 would retain their original solution that would bring $22 and $18 with 11.99 and 11.18 kg N/mo excreted, respectively. Diet 4 would have the best net revenue ($22), followed diet 2 ($20) and then by diet 5 ($18). Diet 1 under unfavorable market conditions and N excretion constraint would be the diet with the worst net return ($11)(Table 3). Therefore under unfavorable market conditions, it would cost $4/mo per cow to reduce 0.38 kg of N excreted with diet 1, $1/cow per mo to reduce 0.35 kg N excreted with diet 2 and $4/mo per cow to reduce 0.46 kg N excreted with diet 3. A restriction of 11.35 kg N/mo excreted (level reached with only diet 5 with favorable market conditions) would result in non-feasible solutions for any of the concentrate diets, meaning that none of them could reach that low amount of N excretion.

CONCLUSIONS

A Markovian linear programming formulation and solution of the dynamic programming of dairy herd economic optimization problem represents a contribution to practical dairy herd decision-making tools applied to the replacement problem. It complements and adds to the value and policy interaction methods commonly used to solve large dynamic programming models. This study found the maximum net revenue for optimal and sub-optimal dairy herd replacement policies for 5 different diets under different price scenarios. The model found consistently: (1) keep pregnant cows regardless of their production level (the net revenue to be realized with the new born and subsequent lactations is always more valuable than a replacement); (2) allow primiparous cows to stay in the herd for more months — and try more services prior to culling — compared with their multiparous herdmates (because of expectation of higher production in late lactation for the former compared with the latter); and (3) allow higher culling rates when economic market conditions determine low milk price, high corn price and low replacement cost. Under favorable market conditions diets with a high proportion of concentrates realize higher net revenues, but under unfavorable market conditions, diets with high forage content or with only alfalfa silage would outperform high concentrate diets. Diets with higher concentrates generated higher levels of N excreted. A sub-optimal solution of the model to limit the N excretion per cow to 12 kg/mo resulted in the diet with the second highest level of concentrate (diet 2) to provide for the highest net revenue under favorable market conditions. With unfavorable market conditions and under the same N excretion restriction, the least concentrate content diet (diet 4) provided the highest net revenue. The implementation of a Markovian linear program is an important advancement for dairy decision-making that provides both robustness and versatility in operations research. The model could become a valuable tool to support economic decision-making of dairy herd management.

ACKNOWLEDGMENTS

The author appreciates the cooperation of AgSource DHI Cooperative Services (http://agsource.crinet.com/page249/DHI) for providing the baseline data used to parameterize reproduction rates, involuntary culling, and mortality rates on Holstein cows. The author appreciates the professional comments and feedback of Michel Wattiaux and 2 anonymous reviewers on an earlier version of the manuscript.

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Table 1. Pregnancy rate by MIL and parity for Wisconsin Holsteins

	Pregnancy Rate (%)	
MIL[1]	Primiparous	Multiparous
2	23.68	21.03
3	18.88	17.95
4	13.63	14.29
5	9.95	11.07
6	7.48	8.75
7	5.73	6.80
8	4.49	5.35
9	4.28	5.27
10	4.58	5.46
11	4.93	5.16
12	4.98	5.41
13	5.17	5.57
14	5.05	5.75
15	5.01	5.49

[1] MIL is month of lactation.

Source: Adapted from 326,000 Holstein lactations (2003-2007) provided by AgSource DHI Cooperative Services.

Table 2. Percentage (%) of ingredients on a dry basis in diets according to lactation stages. Note: all diets had a 2% content of minerals and vitamins.

	Diet 1 (60% concentrate)		
Month of lactation (MIL)	1-3	4-7	8-22
Alfalfa silage	38	48	68
High moisture ear corn	42	40	25
Soybean meal	18	10	5
	Diet 2 (50% concentrate)		
Alfalfa silage	48	58	78
High moisture ear corn	34	33	17
Soybean meal	16	7	3
	Diet 3 (40% concentrate)		
Alfalfa silage	58	68	88
High moisture ear corn	27	25	9
Soybean meal	13	5	1
	Diet 4 (30% concentrate)		
Alfalfa silage	68	88	98
High moisture ear corn	19	9	0
Soybean meal	11	1	0
	Diet 5 (All-forage diet)		
Alfalfa silage	98	98	98
High moisture ear corn	0	0	0
Soybean meal	0	0	0

Source: Adapted from Tessmann et al. (1991).

Table 3. Optimal policy, N excreted and net revenue selected for model according to market conditions, diet and N constraint

Market and Constraint Conditions	Diet[1]	MIL Replacement[2]	N excretion (kg/cow/mo)	Net Revenue ($/cow/mo)
2008 Favorable	1	11	12.56	132.16
Milk $0.40/kg	2	11	12.47	131.79
Corn $0.19/kg	3	11	12.55	116.92
Replacement $2,000	4	11	12.09	105.49
No N constraint	5	12	11.35	79.84
2008 Unfavorable	1	9	12.38	15.06
Milk $0.22/kg	2	9	12.35	21.04
Corn $0.24/kg	3	9	12.46	18.71
Replacement $1,500	4	9	11.99	21.97
No N constraint	5	10	11.18	18.38
2008 Favorable	1	9[3]	12.00	119.84
Milk $0.40/kg	2	9[3]	12.00	126.36
Corn $0.19/kg	3	9[3]	12.00	104.86
Replacement $2,000	4	10	12.00	104.94
N ≤ 12 kg/mo constraint	5	12	11.35	79.84
2008 Unfavorable	1	7[3]	12.00	10.98
Milk $0.22/kg	2	9[3]	12.00	19.88
Corn $0.24/kg	3	8[3]	12.00	14.84
Replacement $1,500	4	9	11.99	21.97
N ≤ 12 kg/mo constraint	5	10	11.18	18.38

[1] As defined in Table 2: Diet 1 had 60% concentrate, diet 2 had 50% concentrate, diet 3 had 40% concentrate, diet 4 had 30% concentrate and diet 5 was all-forage diet. [2] Month of lactation suggested to replace a primiparous open cow; for all other cases the replacement for multiparous occurred 1 MIL less. [3] Herd distribution did not follow a defined pattern after first parity.

Table 4. Herd structure associated with maximum net revenue for second parity, high concentrate diet and 2008 market conditions. Numbers represent the proportion of cows of the entire herd in each specific state defined by parity, mo in lactation and pregnancy status when the model reaches steady state

	Pregnancy Status[1]								
MIL[2]	0	1	2	3	4	5	6	7	8
1	0.023785[3]								
2	0.023188								
3	0.017862	0.004801							
4	0.014336	0.003161	0.004703						
5	0.012213	0.002026	0.003103	0.004452					
6	0.010896	0.001338	0.001992	0.002944	0.004268				
7	0.009999	0.000946	0.001318	0.001894	0.002828	0.004142			
8	0.009351	0.000674	0.000934	0.001255	0.001822	0.002749	0.004068		
9	0.008862	0.000497	0.000667	0.000890	0.001209	0.001773	0.002703	0.004011	
10		0.000465	0.000492	0.000637	0.000859	0.001179	0.001747	0.002669	0.003966
11			0.000461	0.000470	0.000615	0.000838	0.001162	0.001726	0.002642
12				0.000441	0.000455	0.000601	0.000827	0.001150	0.001711
13					0.000427	0.000444	0.000593	0.000819	0.001140
14						0.000417	0.000439	0.000588	0.000813
15							0.000413	0.000435	0.000584
16								0.000409	0.000432
17									0.000407
18									
19									
20									
21									
22									
23									
24									

[1] 0 = open cows and 1 to 8 are months in gestation. [2] MIL is month of lactation. [3] 2.38% of cows in herd enter lactation 2. Similar herd structures were generated for each one of the 15 parities included in the model after each solution.

Figure 1. Representation of probabilistic Markovian processes of cow flow transitions.

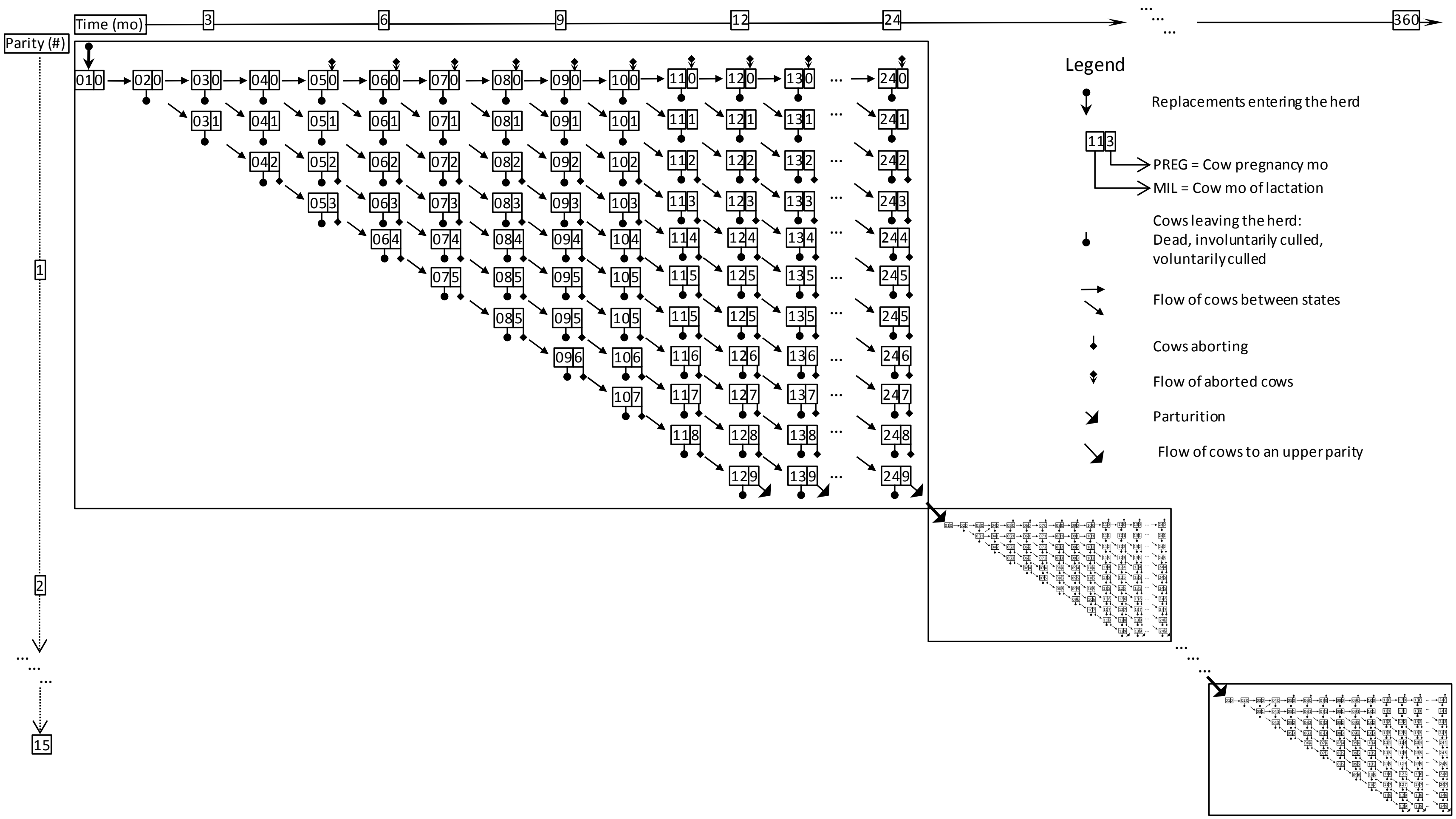
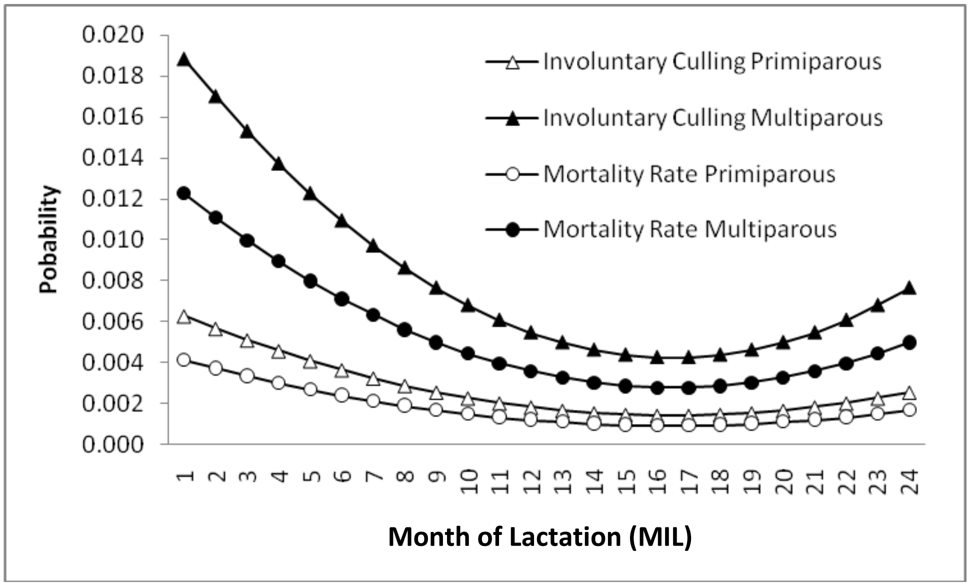


Figure 2. Mortality rate and involuntary culling in Midwest Holsteins



Source: Adapted from 326,000 Holstein lactations (2003-2007) provided by AgSource DHI Cooperative Services. Note: Involuntary culling is assumed to be 3 times the mortality rate.

Figure 3. Proportion (probability) of herd population according to parity (a) and month in lactation (b) when the herd is in steady state for 2008 market conditions and fed the highest concentrate diet (diet 1 on Table 2)