Optimal Crop Planting Schedules and Financial Hedging Strategies Under ENSO-based Climate Forecasts

Farid AitSahlia[1], Chung-Jui Wang[2], Victor E. Cabrera[3], Stan Uryasev[4], and Clyde W. Fraisse[5]

Abstract

This paper investigates the impact of ENSO-based climate forecasts on optimal planting schedules and financial yield-hedging strategies in a framework focused on downside risk. In our context, insurance and futures contracts are available to hedge against yield and price risks, respectively. Furthermore, we adopt the Conditional-Value-at-Risk (CVaR) measure to assess downside risk, and Gaussian copula to simulate scenarios of correlated non-normal random yields and prices. The resulting optimization problem is a mixed 0-1 integer programming formulation that is solved efficiently through a two-step procedure, first through an equivalent linear form by disjunctive constraints, followed by decomposition into sub-problems identified by hedging strategies. With data for a representative cotton producer in the Southeastern United States, we conduct a study that considers a wide variety of optimal planting schedules and hedging strategies under alternative risk profiles for each of the three ENSO phases (Niña, Niño, and Neutral.) We find that the Neutral phase generates the highest expected profit with the lowest downside risk. In contrast, the Niña phase is associated with the lowest expected profit and the highest downside risk. Additionally, yield-hedging insurance strategies are found to vary significantly, depending critically on the ENSO phase and on the price bias of futures contracts.

[1] Risk Management and Financial Engineering (RMFE) Lab., Department of Industrial and Systems Engineering, University of Florida; Email: ise.farid@ufl.edu; URL: www.ise.ufl.edu/farid
[2] RMFE Lab., Dept. of Industrial and Systems Engineering, University of Florida; Email: cjwang@ufl.edu.
[3] Department of Dairy Science, University of Wisconsin-Madison; Email: vcabrera@wisc.edu.
[4] RMFE Lab., Department of Industrial and Systems Engineering, University of Florida; Email: uryasev@ufl.edu; URL: www.ise.ufl.edu/uryasev (corresponding author)
[5] **Agricultural and Biological Engineering, University of Florida; Email: cfraisse@ufl.edu; URL: http://www.agclimate.org/fraisse**

1. Introduction

A typical farming business is mostly concerned with crop yields and the prices at which they are sold at harvest. The source of this concern is rooted in climatic variations. Extreme weather conditions, such as precipitations occurring too soon or too late, or in too little amount, will likely result in low-yield crops, which may not generate enough revenues to offset all crop-related costs. On the other hand, even ideal and timely precipitations can adversely affect profits as abundant crops produced by competing farms may lead to sales price drops.

To hedge against low-yields, the Risk Management Agency (RMA) of the United States Department of Agriculture (USDA) offers three types of crop insurance policies: yield-based insurance, revenue-based insurance, and policy endorsement. A yield-based insurance policy, such as the Actual Production History (APH), insures producers against yield losses due to natural causes. A revenue-based insurance policy, such as Crop Revenue Coverage (CRC), provides revenue protection. Catastrophic Coverage (CAT), a policy endorsement, pays 55% of the price, established annually by RMA, of the commodity on crop yield shortfall in excess of 50%. The cost of crop insurance includes a premium and an administration fee. On the other hand, the uncertainty of commodity prices can be hedged through futures contracts, which are binding agreements between a buyer and a seller for delivery of a particular quantity of a commodity at a specified future date, price, and place. Futures contracts are highly standardized and are traded on exchanges such as the New York Mercantile Exchange (NYMEX). The cost of a futures contract includes commissions and interest foregone on margin deposit. A risk-averse producer may consider using insurance products in conjunction with futures contracts for the best possible outcome. Substantial research efforts have focused on crop risk-hedging using crop insurance and derivative securities such as futures. Poitras (1993) studied a farmer's optimal hedging problem when both futures and crop insurance are available to reduce the uncertainty of price and production. Chambers and Quiggin (2002) examined optimal producer behavior in the presence of area-yield insurance. Mahul (2003) investigated the demand for futures and options to hedge against price risk when crop yield and revenue insurance contracts are available. Coble, Miller, and Zuniga (2004) investigated the effect of crop insurance and loan programs on demand for futures contracts.

1.1 Impact of ENSO climate forecasts on hedging strategies

The El Niño - Southern Oscillation (ENSO) phenomenon is a global event arising from large-scale interactions between the ocean and the atmosphere. El Niño is a disruption of the ocean-atmosphere system in the tropical Pacific that has important consequences on weather around the globe. This condition results in a redistribution of rains with flooding and droughts. Southern Oscillation refers to an oscillation (difference) in the surface pressure (atmospheric mass) between the southeastern tropical Pacific and the Australian-Indonesian regions. When the waters of the eastern Pacific are abnormally warm (an El Niño event) sea level pressure drops in the eastern Pacific and rises in the west. The reduction in the pressure gradient is accompanied by a weakening of the low-latitude. On the other hand, when the east-west barometric pressure gradient is reversed, the eastern Pacific sea surface temperature drops below normal. This is called a "La Niña'' event. Sea surface temperatures within a normal range are called "Neutral". These equatorial Pacific conditions, known as ENSO phases, refer to different seasonal climatic conditions.

As the Pacific sea surface temperatures are statistically predictable, ENSO has become an index for forecasting climate and, consequently, crop yields. Cane, Eshel, and Buckland (1994), Hansen, Hodges, and Jones (1998), Hansen (2002), and Jones et al. (2000) have investigated the connection between ENSO-based climate predictions and crop yields. More recently, some researchers have studied the impacts of the ENSO-based climate information on the selection of optimal crop insurance policies. Cabrera et al. (2006) examined the impact of ENSO-based climate forecast on reducing farm risk with optimal crop insurance strategy. Cabrera, Letson, and Podesta (2007) included the interference of farm government programs on crop insurance hedge under ENSO climate forecast.

1.2 Efficient frontiers, hedging strategies, and ENSO climate forecasts

Given an ENSO-based climate forecast, a farmer should also be able to optimally select a planting schedule to maximize expected revenues. These expected revenues can be further increased when hedging instruments such as crop-yield insurance and commodity futures contracts are available. In this paper we develop a model resulting in an efficient frontier that accounts for the trade-off between the risk attitude of the farmer and the desired expected revenues given ENSO-based climate forecasts. We adopt the Conditional-Value-at-Risk measure (see, e.g., Rockafellar & Uryasev, 2000) to assess the risk-aversion driving the optimal planting schedule and selection of hedging instruments such as insurance and futures contracts. In addition, as the sources of uncertainty for this model, namely the correlated yield and price, are not necessarily normally distributed, we adopt the Gaussian copula approach to capture their joint distribution.

The remainder of this article is organized as follows. Section 2 details the optimization model that we develop. Section 3 briefly reviews the copula approach to capture the joint distribution of correlated random variables and calibrate the model to our data. Section 4 describes the data of a representative cotton producer in the Southeastern United States that we used for our empirical study, which is then summarized in the concluding Section 5.

2. Optimization Model

In this section we set up our optimization model to determine an efficient frontier to capture the risk-reward structure inherent in the interplay between planting and financial hedging decisions on one hand, and yield and price uncertainty on the other. First, we review our measure of risk, Conditional-Value-at-Risk (CVaR), and then define formally our optimization model. To solve it, we express it as a mixed 0-1 integer programming formulation, where we approximate expected values by their sample means. The latter result from a Monte Carlo simulation of scenarios generated on the basis of empirical distributions, which will be further described in our data section 4.

2.1 The Conditional-Value-at-Risk Measure

The seminal work of Markowitz (1952) pointed to the possibility of assessing risk through a statistical value, the variance or standard deviation, which has been widely adopted. One of its drawbacks though is its limitation to random variables with finite variance. In addition, it does not distinguish between favorable outcomes (e.g., higher than a specific value) and unfavorable ones (e.g., less than another specific value). In recent years, the alternative measure of Value-at-Risk (VaR) has been especially popular among those concerned with financial risk management (cf., Jorion 2000). It first gained acceptance at JP Morgan in the early 1990's. Specifically, for 0<α<1, VaR_α evaluates the α^{th}-percentile of a loss distribution, with usually α=.90 or .95. When dealing with a gain distribution, the risk focus is then on negative values and α usually is typically set at .10 or .05. VaR however has undesirable properties, especially for outcomes with non-elliptical distributions. In particular, it may violate the fairly well-accepted principle of diversification; i.e., the VaR of a combination of portfolios may not be smaller than that of each of its elements. This drawback stems primarily from the fact that VaR only considers a specific (high) percentile of a loss distribution without regard to the magnitude of the loss. As a consequence, a variant of VaR, usually labeled *Conditional-Value-at-Risk* (or CVaR), has been steadily gaining acceptance in the past few years (cf. Artzner et al. (1999), Szegö 2002). Formally, if ξ is loss random variable and $\alpha \in (0,1)$, we define

$$CVaR_\alpha(\xi) = E\left[\xi \mid \xi \geq VaR_\alpha(\xi)\right].$$

Another reason for the adoption of CVaR in the efficient frontier context is practical. In particular, Rockafellar and Uryasev (2000) showed that CVaR constraints in optimization problems can be formulated as linear constraints. This linear property is crucial to the formulation of our model as a mixed 0-1 linear programming problem that can be efficiently solved.

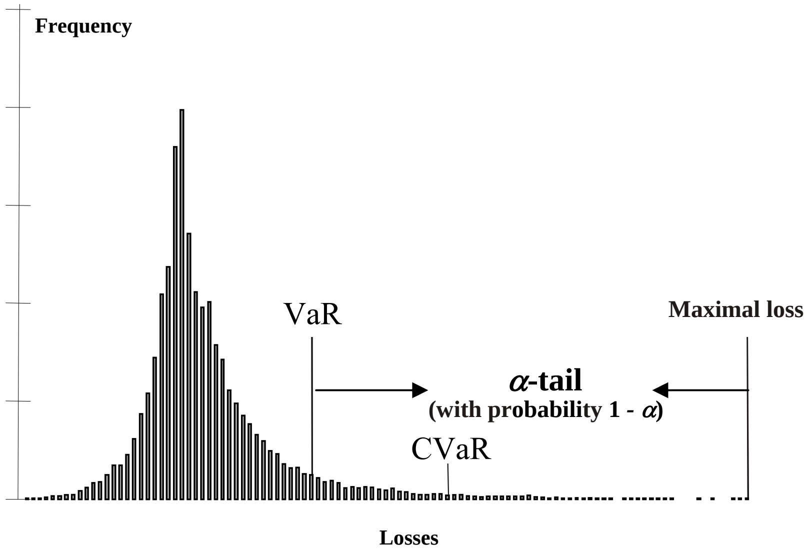


Figure 1. VaR and CVaR associated with a loss distribution

2.2 A Profit Maximization Model

We consider a farmer who plans to grow crops on a farmland of Q acres in a given year. There are K possible types of crops and more than one crop can be planted at a time during that year. For crop k, $1 \leq k \leq K$, there are T_k potential planting dates, and I_k available insurance policies. The decision variable x_{kti} represents the acreage of crop k planted on date t, $1 \leq t \leq T_k$, with insurance policy i , $i \in I_k$. Let η_k , another decision variable, represent the hedge position (in pounds) of crop k in a futures contract. We assume for each crop that harvest will occur in one season and that it will be sold at one price, irrespective of when it was planted (and thus when picked during the harvest season.) Additionally, we assume that the corresponding random yield Y_{kt} (for crop k planted at time t) and price P_k have joint distributions specific to each of the three ENSO phases. The objective is to maximize the expected value of the profit function

$\sum_{k=1}^{K} f(\tilde{x}_k, \eta_k)$ resulting from the planting and insurance strategies $\tilde{x}_k = (x_{kti})_{t,i}$, and futures hedging strategy η_k for crops $k = 1, \ldots, K$, with

(1) $$f(\tilde{x}_k, \eta_k) = f^P(\tilde{x}_k) + f^I(\tilde{x}_k) + f^F(\eta_k),$$

where $f^P(\tilde{x}_k)$, $f^I(\tilde{x}_k)$, and $f^F(\eta_k)$ are the corresponding random profits resulting from production, crop insurance, and futures contracts, respectively, and are defined next. For production, we assume that

(2) $$f^P(\tilde{x}_k) = P_k \sum_{t=1}^{T_k} \left(Y_{kt} \sum_{i \in I_k} x_{kti} \right) - (C_k - S_k) \sum_{t=1}^{T_k} \sum_{i \in I_k} x_{kti},$$

where C_k is the unit production cost for crop *k* and S_k the corresponding subsidy rate (per unit of production.) For insurance, three types of policies are considered: Actual Production History (APH), Crop Revenue Coverage (CRC), and Catastrophic Coverage (CAT). For APH, a farmer selects (i) the insured yield, a percentage α between 50 to 75 percent with five-percent increments of the expected yield $E(Y_k)$, and (ii) the election price, a percentage β, between 55 and 100 percent, of the price $\hat{P}_k$ established annually by RMA. If we let I_{APH} denote the set of all APH insurance policies available to this farmer, the indemnity for any crop 1≤k≤K with insurance policy $i \in I_{APH}$ is then defined as

(3) $$D_{ki} = \max\left[\sum_{t=1}^{T_k} (\alpha_i E(Y_k) - Y_{kt}) x_{kti}, 0 \right] \times \beta_i \hat{P}_\mathrm{k},$$

where α_i and β_i are the corresponding insured yield and election price percentages, respectively. For CRC, the producer selects a percentage of coverage level γ between 50 and 75 percent. With I_{CRC} defined as the set of all available CRC policies, the indemnity of an insurance policy $i \in I_{CRC}$ for crop 1≤k≤K is

$$D_{ki} = \max\left[\gamma_i \sum_{t=1}^{T_k} E(Y_k)x_{kti} \times \max\left[P_k^b, P_k\right] - \sum_{t=1}^{T_k} Y_{kt} x_{kti} P_k, 0\right], \tag{4}$$

where P_k^b is its base (early-season) price and γ_i the corresponding (percentage) coverage level. In other words, a policy $i \in I_{CRC}$ guarantees revenue $\gamma_i \sum_{t=1}^{T_k} E(Y_k)x_{kti} \times \max\left[P_k^b, P_k\right]$ for crop *k.* Finally, CAT insurance pays 55 % of the established price of the commodity on crop losses in excess of 50 %. Therefore, for crop 1≤k≤K, the indemnity of a CAT insurance policy $i \in I_{CAT}$, where I_{CAT} is the set of all such policies, is defined

$$D_{ki} = \max\left[\sum_{t=1}^{T_k} \left(0.5E(Y_k) - Y_{kt}\right)x_{kti}, 0\right] \times 0.55P_k \quad . \tag{5}$$

For crop 1≤k≤K and I_k its associated set of available insurance policies, let R_{ki} denote the corresponding cost per acreage. Then the resulting total profit due to planting and insurance strategy $\tilde{x}_k = \left(x_{kti}\right)_{1\le t\le T_k, i\in I_k}$ is

$$f^I\left(\tilde{x}_k\right) = \sum_{i\in I_k}\left(D_{ki} - R_{ki}\sum_{t=1}^{T_k} x_{kti}\right). \tag{6}$$

As for the futures strategy, we assume that the farmer selects the hedge position η_k (in pounds) for crop *k* at the same time that strategy $\tilde{x}_k = \left(x_{kti}\right)_{1\le t\le T_k, i\in I_k}$ is determined (i.e.; at the beginning of the planting period when the ENSO forecasts are available.) This is a simplifying feature and we could as well incorporate, at the cost of slighter complexity, a futures strategy that is also time dependent. Then the payoff at harvest of a futures contracts strategy η_k for crop k is

(7) $$\pi_k^F = (F_k - f_k)\eta_k ,$$

where F_k and f_k are the corresponding futures prices at the start of the planting season and harvest, respectively. It should be noted that f_k is not exactly the price at which crop k is sold at harvest, which is P_k. The resulting random difference $P_k - f_k$, termed basis, can be estimated with actual historical prices. The cost C_k^F of a futures contracts strategy η_k includes commissions and interest foregone on margin deposit to account for the so-called marking to market of futures, leading to the profit function

(8) $$f^F(\eta_k) = \pi_k^F - C_k^F .$$

In summary, we now have in (2), (6), and (8) explicit forms for the profit function (1) that results from the planting and insurance strategies $\tilde{x}_k = (x_{kti})_{t,i}$, and futures hedging strategy η_k for crops $k = 1,\ldots,K$. The objective is therefore to maximize

(9) $$E\left(\sum_{k=1}^{K} f(\tilde{x}_k,\eta_k)\right) = \sum_{k=1}^{K}\left(E\left[f^P(\tilde{x}_k)\right] + E\left[f^I(\tilde{x}_k)\right] + E\left[f^F(\eta_k)\right]\right),$$

where for $k = 1,\ldots,K$ each expectation is taken over the joint density of Y_k and P_k. Additionally, we capture the producer's risk attitude through a set of CVaR constraints, namely, for $k = 1,\ldots,K$, and given α and U,

(10) $$CVaR_\alpha\left(L(\tilde{x}_k,\eta_k)\right) \le U ,$$

where $L(\widetilde{x}_k, \eta_k) = -f(\widetilde{x}_k, \eta_k)$ and

$$(11) \qquad CVaR_\alpha(L(\widetilde{x}_k, \eta_k)) = E[L(\widetilde{x}_k, \eta_k) \mid L(\widetilde{x}_k, \eta_k) \geq \zeta_\alpha(L(\widetilde{x}_k, \eta_k))],$$

with $\zeta_\alpha(L(\widetilde{x}_k, \eta_k))$ defined as the α-quantile of the loss distribution $L(\widetilde{x}_k, \eta_k)$ and the expectation taken again over the joint distribution of Y_k and P_k.

2.3 A Mixed 0-1 Integer Programming Formulation

The optimization problem with objective function (9) and side constraints of the form (11) is now complete. We further simplify it by appealing to results of Rockafellar & Uryasev (2000) who provide an alternative expression for the CVaR quantity (11) and indicate that it is robust to sample mean approximations of expected values. In particular, they show that the CVaR constraint (11) can be replaced by the linear expressions

$$(12) \qquad \zeta_\alpha(L(\widetilde{x}_k, \eta_k)) + \frac{1}{J(1-\alpha)} \sum_{j=1}^{J} z_j \leq U,$$

$$(13) \qquad \sum_{k=1}^{K} \sum_{t=1}^{T_k} \sum_{i=1}^{I_k} L(x_{kti}, \eta_k) - \zeta_\alpha(L(x_{kti}, \eta_k)) \leq z_j,$$

$$(14) \qquad 0 \leq z_j,$$

where z_j, $1 \leq j \leq J$, are supplementary variables and J is the number of scenarios (samples) generated via Monte Carlo on the basis of the joint distribution of Y_k and P_k. Furthermore, if we let Y_{ktj} denote the j^{th} realized (sampled) yield (pound per acre) of crop k planted on date t, and P_{kj} denote the j^{th} realized (sampled) cash price (dollar per pound) for crop k at the time the crop will be sold, we can replace all expectations in (9) as follows:

(15) $$Ef^{P}(x_{kti}) \approx \frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T_k}(Y_{ktj}P_{kj} - C_k + S_k)\sum_{i=1}^{I_k} x_{kti} ,$$

(16) $$Ef^{I}(x_{kti}) \approx \frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{i=1}^{I_k} D_{kij} - R_{ki}\sum_{t=1}^{T_k} x_{kti} ,$$

(17) $$Ef^{F}(\eta_k) \approx \frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{K}(\pi_{kj}^{F} - C_k^{F}),$$

where, depending on the contract type i, D_{kij} is one of the expressions (3), (4), or (5), with Y_{kt} replaced by Y_{ktj} and $E(Y_k)$ by $\overline{Y}_k = \frac{1}{J}\sum_{t=1}^{T_k}\sum_{j=1}^{J} Y_{ktj}$, and $\pi_{kj}^{F} = (F_k - f_{kj})\eta_k$, with f_{kj} being the j^{th} realized (sampled) futures price of crop k at harvest time.

As only one insurance policy can be selected for each crop k, we introduce binary variables ϑ_{ki} subject to the conditions

(18) $$\sum_{t=1}^{T_k} \theta_{kti} \le Q \cdot \theta_{ki} \qquad i \in I_k ,$$

(19) $$\sum_{i=1}^{I_k} \theta_{ki} = 1 ,$$

where Q is the total acreage available for planting and

$$\theta_{ki} = \begin{cases} 1 & \text{if crop } k \text{ is insured by policy } i, \\ 0 & \text{otherwise.} \end{cases}$$

In addition, we may impose

(20) $$\sum_{k=1}^{K}\sum_{t=1}^{T_k}\sum_{i=1}^{I_k} x_{kti} = Q .$$

The equality in this constraint can be replaced by an inequality $(\le)$ to represent a farmer's choice to not grow the crops when production is not profitable.

Note that the maximum objective function contains indemnities D_{kij} that include a *max* term shown in expressions (3), (4), and (5). To implement the model as a mixed 0-1 linear problem, we transform these equations to an equivalent linear formulation by disjunctive constraints (Nemhauser and Wolsey, 1999). For example, equation (3), $D_{kij} = \max\left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}, 0\right] \times \beta_i P_k$, can be represented by the set of mixed 0-1 linear inequality constraints

$$
\begin{aligned}
& D_{kij} \geq 0, \\
& D_{kij} \geq \left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}\right] \times \beta_i P_k, \\
& D_{kij} \leq \left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}\right] \times \beta_i P_k + MZ_{kij}, \\
& D_{kij} \leq \left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}\right] \times \beta_i P_k + M(1\text{-}Z_{kij}), \\
& \left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}\right] \times \beta_i P_k \leq M(1\text{-}Z_{kij}), \\
& \left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}\right] \times \beta_i P_k \geq -MZ_{kij}.
\end{aligned}
\tag{21}
$$

where M is a large number and Z_{kij} is a 0-1 variable. Equations (4) and (5) can be transformed into a set of mixed 0-1 linear constraints in the same way. Consequently, the optimal crop production and hedging problem can be formulated as a mixed 0-1 linear programming problem.

2.4 Further decomposition for efficient computations

Although the mixed 0-1 linear programming problem can be solved with optimization software, its computation time increases exponentially as the problem becomes large. To improve computational efficiency, the original problem is decomposed into sub-problems in which each crop is insured by a specific insurance policy. The original problem contains K types of crops, and for the k^{th} type of crop there are I_k eligible insurance policies. Therefore, the number of the sub-problems is equal to the number of all possible insurance combinations of the K crops, $\prod_{k=1}^{K} I_k$. The sub-problems are the same as the original problem except that the index *i's* are fixed and equations (3) and (11) are removed. Solving the sub-problems gives the optimal production strategy and futures hedge amount under a specific combination of insurance policies for K crops. The solution of the sub-problem with the highest optimal expected profit gives the optimal solution of the original problem. The optimal production strategy and futures hedge position are provided from the said sub-problem solution and the optimal insurance coverage is the specific insurance combination of the sub-problem.

3. Model Calibration and Simulation: A Copula Approach

To investigate the impact of ENSO-based climate forecast on the optimal production and risk management decisions, we calibrate the joint distribution of crop yields and price for individual ENSO phases based on the historical yields and prices of the years classified to the ENSO phase. Then, random yield and price scenarios associated with the ENSO phase are generated by Monte Carlo simulation. The correlations between yields in various planting dates and between yields and price are considered and modeled by copula method. Copulas are functions describing dependencies among variables and provide a way to create distributions to model correlated multivariate data. Sklar's theorem (Sklar 1959) states that given a joint distribution function F on R^n with marginal distribution F_i, there is a copula function C such that for all $x_1,...,x_n$ in R,

$$F(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n)).$$

Furthermore, if F_i are continuous then C is unique. Conversely, if C is a copula and F_i are distribution functions, then F as defined by the previous expression is a joint distribution function with margins F_i. In this paper, we apply the Gaussian copula method to generate the correlated non-normal multivariate distribution. The Gaussian copula is given by:

$$C_\rho(F_1(x_1),...,F_n(x_n)) = \Phi_{n,\rho}(\Phi^{-1}(F_1(x_1)),...,\Phi^{-1}(F_n(x_n))),$$

which maps the observed variable x_i, i.e. yield or price, into a new variable y_i using the transformation

(22) $$y_i = \Phi^{-1}\left[F_i(x_i)\right],$$

where $\Phi_{n,\rho}$ is the joint distribution function of a multivariate Gaussian vector with a mean of zero and correlation matrix ρ. Φ is the distribution function of a standard Gaussian random variable. In moving from x_i to y_i the observation from the assumed distribution F_i is mapped into a standard normal distribution Φ on a percentile to percentile basis.

We use the rank correlation coefficient ρ_s, Spearman's rho, to calibrate the Gaussian copula to the historical data. For *n* pairs of bi-variate random samples (X_i, X_j), define $R_i = rank(X_i)$ and $R_j = rank(X_j)$. Spearman's sample rho is given by

$$\rho_s = 1 - 6\frac{\sum_{i=1}^{n}\left(R_i - R_j\right)}{n(n^2-1)}.$$

Spearman's rho measures the association only in terms of ranks. The rank correlation is preserved under the monotonic transformation in (22). Furthermore, there is a one-to-one mapping between the rank correlation coefficient ρ_s and the linear correlation coefficient ρ for bi-variate normal random variables (y_1, y_2) (Kruksal 1958):

$$\rho_s(y_1, y_2) = \frac{6}{\pi}\arcsin\frac{\rho(y_1, y_2)}{2}.$$

To generate correlated multivariate non-normal random variables with margins F_i and Spearman's rank correlation matrix ρ_s, we generate the random variables y_i's from the multivariate normal distribution $\Phi_{n,\rho}$ with linear correlation

$$\rho = 2\sin\left(\frac{\pi\rho_s}{6}\right),$$

by Monte Carlo simulation. The actual outcomes x_i's can be mapped from y_i's using the transformation

$$x_i = F_i^{-1}\left[\Phi(y_i)\right].$$

4. Case Study

Following Cabrera et al. (2006), we consider a representative farmer who grows cotton in a non-irrigated farm of 100 acres in Jackson County, Florida. Dothan Loamy Sand, the dominant soil type in the region, is assumed. The farmer may buy futures contracts on the New York Board of Trade and/or purchase crop insurance to hedge against crop yield and price risks. Three types of crop insurance, Actual Production History (APH), Crop

Revenue Coverage (CRC), and Catastrophic Coverage (CAT), are available for cotton yield-hedging. For each crop, the farmer may select only one insurance policy or opt for no insurance at all. For APH, the eligible coverage levels of yield are from 65% to 75% with 5% increments, and the election price is assumed to be 100% of the established price. In addition, the available coverage levels of revenue for CRC are from 65% to 85% with 5% increments.

For numerical implementation, we used Historical climate data from 1960 to 2003. ENSO phases during this period included 11 years of El Niño, 9 years of La Niña, and the remaining 24 years of Neutral, according to the Japan Meteorological Agency's definitions (Japan Meteorological Agency 1991).

Table 1. Historical Years Associated with ENSO Phases, 1960-2003

EL Niño		Neutral				La Niña	
1964	1987	1960	1975	1984	1994	1965	1989
1966	1988	1961	1978	1985	1995	1968	1999
1970	1992	1962	1979	1986	1996	1971	2000
1973	1998	1963	1980	1990	1997	1972	
1977	2003	1967	1981	1991	2001	1974	
1983		1969	1982	1993	2002	1976	

Cotton yields during the years 1960 through 2003 were simulated using the CROPGRO-Cotton model (Messina, Jones, and Fraisse 2005) in the Decision Support System for Agrotechnology Transfer (DSSAT) v4.0 (Jones et al., 2003) based on the historical climate data collected at Chipley weather station. The input for the simulation model followed the current management practice of variety, fertilization, and planting dates in the region. More specifically, a medium to full season Delta & Pine Land® variety

(DP55), 110 kg/ha Nitrogen fertilization in two applications, and four planting dates, 16 Apr, 23 Apr, 1 May, and 8 May, were included in the yield simulation, which was further stochastically re-sampled to produce a series of synthetically generated yields following the historical distributions (for more details see Cabrera et al. 2006).

It was assumed that the cotton would be harvested and sold in December. Therefore, December cotton futures contracts were used to hedge price risk. Historical settlement prices of December futures contracts on the last trading date from 1960 to 2003 were collected from the New York Board of Trade. The statistics and rank correlation coefficient, Spearman's rho matrix of yields, and futures prices are summarized in table 2. They show that crop yields for different planting dates are highly correlated and that correlation of yields decreases when the two corresponding planting dates are farther apart. In addition, a negative correlation between yields and futures price is found in the El Niño and Neutral phases, but not in La Niña.

We further estimated local bases (cf. (7)) using monthly historical data on average cotton prices received by Florida farmers from the USDA National Agricultural Statistical Service (1979 to 2003), which were assumed to be the local cash prices for cotton. We estimated the historical local basis by subtracting the futures price from the local cash price. Using the *Input Analyzer* in the simulation software *Arena*, it was found that the best fitted distribution based on minimum square error method was a beta distribution with probability density function, -0.13+0.15× BETA (2.76, 2.38).

Table 2. Marginal Distributions and Rank Correlation Coefficient Matrix of Yields of Four Planting Dates and Futures Price for the Three ENSO Phases

ENSO	Variable	Statistics of Marginal Distribution		Rank Correlation Coefficient Matrix Spearman's rho				
		Mean	Standard Deviation	Yield (4/16)	Yield (4/23)	Yield (5/1)	Yield (5/8)	Futures Price
El Niño	Yield on 4/16	815.0	71.7	1.00	0.93	0.75	0.74	-0.36
	Yield on 4/23	804.6	79.4	0.93	1.00	0.63	0.57	-0.23
	Yield on 5/1	795.4	99.8	0.75	0.63	1.00	0.75	-0.22
	Yield on 5/8	793.7	79.1	0.74	0.57	0.75	1.00	-0.42
	Futures Price	0.5433	0.1984	-0.36	-0.23	-0.22	-0.42	1.00
Neutral	Yield on 4/16	808.9	108.8	1.00	0.84	0.77	0.62	-0.16
	Yield on 4/23	818.4	100.6	0.84	1.00	0.75	0.64	-0.28
	Yield on 5/1	825.8	86.2	0.77	0.75	1.00	0.75	-0.01
	Yield on 5/8	824.5	68.0	0.62	0.64	0.75	1.00	-0.19
	Futures Price	0.5699	0.1872	-0.16	-0.28	-0.01	-0.19	1.00
La Niña	Yield on 4/16	799.1	99.8	1.00	0.97	0.67	0.60	0.13
	Yield on 4/23	790.7	85.3	0.97	1.00	0.73	0.68	0.20
	Yield on 5/1	793.9	90.6	0.67	0.73	1.00	0.97	-0.13
	Yield on 5/8	809.3	94.1	0.60	0.68	0.97	1.00	-0.08
	Futures Price	0.4669	0.1851	0.13	0.20	-0.13	-0.08	1.00

The Gaussian copula was calibrated based on the sample rank correlation coefficient matrix for the three ENSO phases. For each ENSO phase, 2,000 scenarios of correlated random yields and futures price were sampled by Monte Carlo simulation, based on the Gaussian copula and the empirical distributions of yields and futures price. Further, the basis was simulated and the local cash price was calculated from the futures price and basis.

The futures commissions and opportunity cost of margin was assumed to be \$0.003 per pound, the production cost of cotton was \$464 per acre, and the subsidy for cotton in Florida was \$349 per acre. Finally, the parameters of crop insurance are listed in table 3.

Table 3. Parameters of Crop Insurance (2004) Used in the Farm Model Analysis

Crop Insurance Parameters	Values
APH premium 65%~75%	\$19.5/acre ~\$38/acre
CRC premium 65%~85%	\$24.8/acre~\$116.9/acre
Established Price for APH	\$0.61/lb
Average yield	814 lb/acre

Source: www.rma.usda.gov

4.1 Results and Discussion

This section reports the results of optimal planting schedules and hedging strategies for the three predicted ENSO phases. First, optimal decisions when crop insurance is the only hedging instrument are investigated. Then the best solutions when both insurance and futures contracts are available are analyzed.

Optimal production with only crop insurance coverage

Table 4 shows the optimal planting schedule and hedging strategy considering crop insurance as the only hedging instrument for each ENSO phase, with various 90%CVaR upper bounds ranging from -\$20,000 to -\$2,000. Since the indemnity of CRC depends on the futures price, it is assumed that the futures market is unbiased, i.e., $F = Ef$, where F is the futures price at planting time and f is the random futures price at harvest time. Table 4 shows that ENSO phases affect the expected profit and the feasible region of the downside risk. A Neutral year has the highest expected profit and lowest downside loss.

In contrast, a “La Niña” year has the lowest expected profit and highest downside loss. Additionally, when the upper bound of 90%CVaR constraint is lower than a specific value, which depends on the ENSO phase, the 65%CRC and 70%CRC crop insurance policies are desirable for the optimal hedging strategy in all ENSO phases. In contrast, the APH insurance policy is not desirable for any ENSO phase with 90%CVaR upper bounds. Furthermore, risk can be managed by changing the planting schedule. The last two rows associated with the Neutral phase show that planting 100 acres on date 3 provides a 90%CVaR of -$6,000, which can be reduced to -$8,000 by planting 85 acres on date 3 and 15 acres on date 4. Finally, changing both the insurance coverage and the planting schedule may reduce the downside risk. In La Niña phase, planting 100 acres on date 4 provides a 90%CVaR of -$4,000, which can be reduced to -$10,000 by purchasing a 65%CRC insurance policy and shifting the planting date from date 4 to date 1.

Hedging with crop insurance and futures contracts

The hedge ratio of the futures contract is defined as the hedge position in the futures contract divided by the expected production. We first illustrate the optimal hedging strategies and the optimal planting schedule. Then, the performance of the optimal hedge and planting strategies are gauged via the efficient frontier of expected profit versus CVaR risk. Table 5 shows the optimal insurance policy and futures hedge ratio associated with different 90%CVaR upper bounds for the three ENSO phases. When the futures price is unbiased or positively biased, the futures contract is the only desirable instrument for crop risk management and no insurance policy is needed in the optimal hedging strategy. On the other hand, when the futures price is negatively biased, the optimal hedging strategy includes 65%CRC (or 70%CRC in some cases) insurance policies and a futures contract in the El Niño phase for all feasible 90%CVaR upper bounds. In the Neutral phase, the optimal hedging strategy consists of the 70%CRC insurance policy and a futures contract for all CVaR upper bounds with -10% biased futures prices as well as for CVaR upper bounds between -18000 and -14000 with -5% biased futures prices. In addition, no insurance policy is desirable under La Niña phase.

Table 4. Optimal Insurance and Production Strategies for Each Climate Scenario Under Various 90% CVaR Tolerance Levels

ENSO Phases	90%CVaR Upper Bound	Optimal Expected Profit	Optimal Insurance Strategy	Optimal Planting Schedule			
				Date1	Date2	Date3	Date4
El Niño	<-18000	infeasible					
	-16000	28364	CRC70%	100	0	0	0
	-14000 to -4000	28577	CRC65%	100	0	0	0
	>-2000	28691	No	100	0	0	0
Neutral	<-20000	infeasible					
	-18000	31149	CRC70%	0	0	100	0
	-16000 to -10000	31240	CRC65%	0	0	100	0
	-8000	31779	No	0	0	85	15
	>-6000	31793	No	0	0	100	0
La Niña	<-12000	infeasible					
	-10000 to -6000	20813	CRC65%	100	0	0	0
	>-4000	21572	No	0	0	0	100

Note: Negative CVaR upper bounds represent profits. Four planting dates: ‘Date1’ = April 16, ‘Date2’ = April 23, ‘Date3’ = May 1, ‘Date4’ = May 8. “No” stands for no insurance.

Table 5. Optimal Insurance Policy and Futures Hedge Ratio Under Biased Futures Prices

Bias		-10%		-5%		0%		5%		10%	
ENSO Phases	90% CVaR Upper Bound	Ins.	H.R	Ins.	H.R	Ins.	H.R	Ins.	H.R	Ins.	H.R
El Niño	-28000							x		x	
	-24000			x		x		No	1.00	No	1.24
	-20000	x		65CRC	0.48	No	0.70	No	1.33	No	1.57
	-16000	70CRC	0.20	65CRC	0.15	No	0.52	No	1.60	No	1.87
	-12000	65CRC	-0.01	65CRC	-0.07	No	0.35	No	1.86	No	2.16
	-8000	65CRC	-0.20	65CRC	-0.24	No	0.19	No	2.11	No	2.44
	-4000	65CRC	-0.39	65CRC	-0.41	No	0.02	No	2.35	No	2.73
	0	65CRC	-0.58	65CRC	-0.58	No	0.00	No	2.60	No	3.01
Neutral	-30000									x	
	-28000							x		No	1.06
	-24000			x		x		No	1.19	No	1.42
	-20000	x		No	0.66	No	0.60	No	1.46	No	1.68
	-18000	70CRC	0.28	70CRC	0.12	No	0.50	No	1.57	No	1.80
	-16000	70CRC	0.10	70CRC	-0.02	No	0.40	No	1.68	No	1.92
	-14000	70CRC	-0.04	70CRC	-0.12	No	0.30	No	1.79	No	2.04
	-12000	70CRC	-0.16	No	0.20	No	0.20	No	1.89	No	2.15
	-8000	70CRC	-0.39	No	0.00	No	0.01	No	2.10	No	2.38
	-4000	70CRC	-0.62	No	-0.21	No	0.00	No	2.30	No	2.59
	0	70CRC	-0.86	No	-0.42	No	0.00	No	2.49	No	2.81
La Niña	-20000							x		x	
	-16000			x		x		No	1.04	No	1.23
	-12000	x		No	0.67	No	0.55	No	1.33	No	1.51
	-8000	No	0.33	No	0.27	No	0.23	No	1.57	No	1.77
	-4000	No	-0.10	No	-0.08	No	0.00	No	1.80	No	2.02
	0	No	-0.49	No	-0.41	No	0.00	No	2.02	No	2.27

Note: Negative CVaR upper bounds represent profits. "Ins." stands for Insurance coverage. "H.R." stands for hedge ratio. "No" stands for no insurance. "x" represents infeasible.

Mahul (2003) showed that the hedge ratio contains two parts: a pure hedge component and a speculative component. The pure hedge component corresponds to the hedge ratio associated with unbiased futures price. A positively biased futures price causes the farmer to select a long speculative position and a negatively biased futures price implies a short speculative position. Therefore, the optimal futures hedge ratio under positively (negatively) biased futures prices should be higher (lower) than that under the unbiased futures price.

However, the optimal hedge ratios in Table 5 do not agree with this conclusion when the futures price is negatively biased. The optimal hedge ratios in the La Niña phase illustrate how optimal futures hedge ratios change with the bias of the futures price. Figure 2 shows the bias of futures price versus the optimal hedge ratio curves associated with different 90% CVaR upper bounds in the La Niña phase. When the CVaR constraint is not strict (i.e. the upper bound of 90%CVaR equals zero,) the optimal hedge ratio curve follows the pattern claimed in Mahul (2003). The hedge ratio increases (decreases) with the positive (negative) bias of futures price in a decreasing rate. However, when the

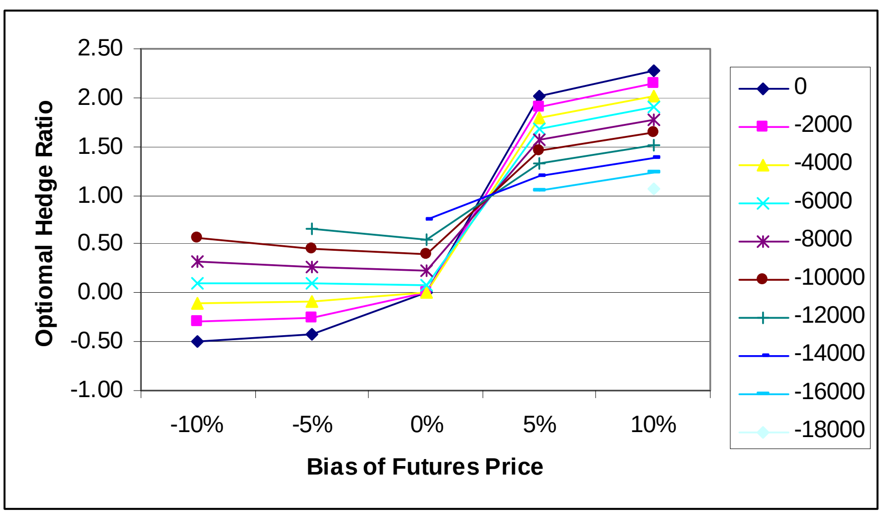


Figure 2. Bias of futures price versus the optimal hedge ratio curves associated with different 90% CVaR upper bounds in the La Niña phase

CVaR constraint becomes stricter (i.e., the CVaR upper bound equals -$8,000), the optimal hedge ratio increases not only with the positive bias but also with the negative one. This is because the higher negative bias of futures price implies a heavier cost (loss) is involved in the futures hedge. It makes the CVaR constraint become stricter, requiring a higher pure hedge component to satisfy the constraint. The net change of the optimal hedge ratio, including an increment in the pure hedge component and a decrement in the speculative component, depends on the loss distribution, the CVaR upper bound, and the bias of futures price.

Table 6. Optimal Planting Schedule for Different Biases of Futures Price in the Three ENSO Phases

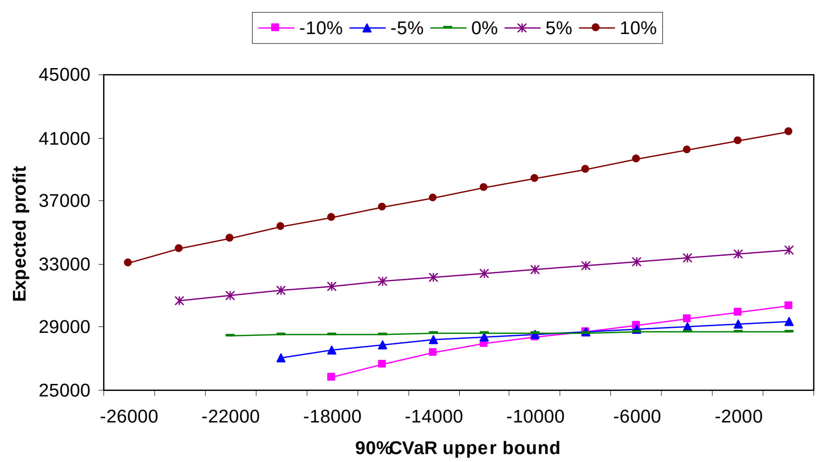
	90%CVaR upper bound	El Niño				Neutral				La Niña			
Bias		Date1	Date2	Date3	Date4	Date1	Date2	Date3	Date4	Date1	Date2	Date3	Date4
	-20000		x				x						
	-18000	100	0	0	0	0	0	97	3				
-10%	-16000	100	0	0	0	0	0	100	0				
	-12000	100	0	0	0	0	0	100	0		x		
	0 to -8000	100	0	0	0	0	0	100	0	0	0	0	100
	-24000		x				x						
	-20000	100	0	0	0	0	0	39	61				
	-18000	100	0	0	0	0	0	100	0				
-5%	-16000	100	0	0	0	0	0	100	0				
	-14000	100	0	0	0	0	0	100	0		x		
	-12000	100	0	0	0	0	0	44	56	0	0	0	100
	-8000	100	0	0	0	0	0	85	15	0	0	0	100
	-4000	100	0	0	0	0	0	92	8	0	0	0	100
	0	100	0	0	0	0	0	93	7	0	0	0	100
	-24000		x				x						
0%	-20000	100	0	0	0	0	0	100	0				
	-16000	100	0	0	0	0	0	100	0		x		
	0 to -12000	100	0	0	0	0	0	100	0	0	0	0	100
	-28000		x				x						
	-24000	100	0	0	0	0	0	47	53				
5%	-20000	100	0	0	0	0	0	68	32		x		
	-16000	100	0	0	0	0	0	92	8	38	0	0	62
	-12000	100	0	0	0	0	0	100	0	35	0	0	65
	-8000	100	0	0	0	0	0	100	0	33	0	0	67

	-4000	100	0	0	0	0	0	100	0	38	0	0	62
	0	100	0	0	0	0	0	100	0	37	0	0	63
	-32000						x						
	-28000		x			0	0	31	69				
	-24000	100	0	0	0	0	0	54	46				
	-20000	100	0	0	0	0	0	75	25		x		
10%	-18000	100	0	0	0	0	0	87	13	33	17	0	50
	-16000	100	0	0	0	0	0	96	4	45	0	0	55
	-12000	100	0	0	0	0	0	100	0	47	0	0	53
	-8000	100	0	0	0	0	0	100	0	54	0	0	46
	-4000	100	0	0	0	0	0	100	0	59	0	0	41
	0	100	0	0	0	0	0	100	0	63	0	0	37

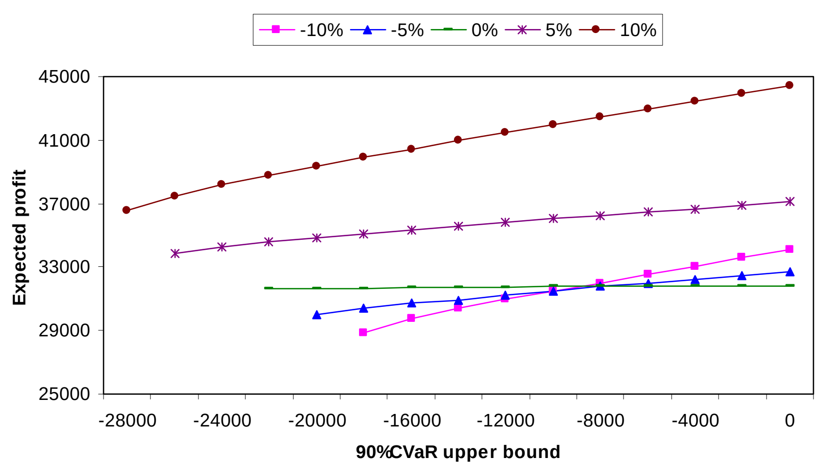
Note: Negative CVaR upper bounds represent profits. Four planting dates: 'Date1' = April 16, 'Date2' = April 23, 'Date3' = May 1, 'Date4' = May 8. "x" represents infeasible.

Table 6 shows the optimal planting schedules for different biases of futures price in the three ENSO phases. For El Niño phase, the optimal planting schedule (i.e., planting 100 acres on date 1) was not affected by the biases of futures price and the 90%CVaR upper bounds. For the Neutral phase, however, the optimal planting strategy was to plant on date 3 and/or date 4, depending on the 90% CVaR upper bounds. More specifically, date 3 is the optimal planting date for all risk tolerance levels under unbiased futures market. When futures prices are positively biased, the lower the 90%CVaR upper bounds (i.e., the stricter the CVaR constraint), the more planting acreages moved to date 4 from date 3. This result expresses the fact that there is no insurance coverage involved in the optimal hedging strategy. With negatively biased futures prices, the optimal planting schedule has the same pattern as positively biased futures markets but is affected by the existence of insurance coverage in the optimal hedging strategy. For example, when the 90%CVaR upper bounds are within the range of -$18,000 and -$14,000 under a -5% biased futures price, the optimal planting acreage on date 4 goes down to zero due to a 75%CRC in the optimal hedging strategy. For La Niña, the optimal planting schedule is to plant 100 acres on date 4 when future prices are unbiased or negatively biased. When the futures price is negatively biased, the stricter the CVaR constraint, the more planting acreage is shifted from date 4 to date 1. For deep positively biased futures price, together with a strict CVaR constraint (i.e., 10% biased futures price and -$18,000 90%CVaR upper bound), the optimal planting schedule includes date 1, 2, and 4.

Efficient frontier in El Nino phase under various biased futures price



Efficient frontier in Neutral phase under various biased futures price



Efficient frontier in La Nina phase under various biased futures price

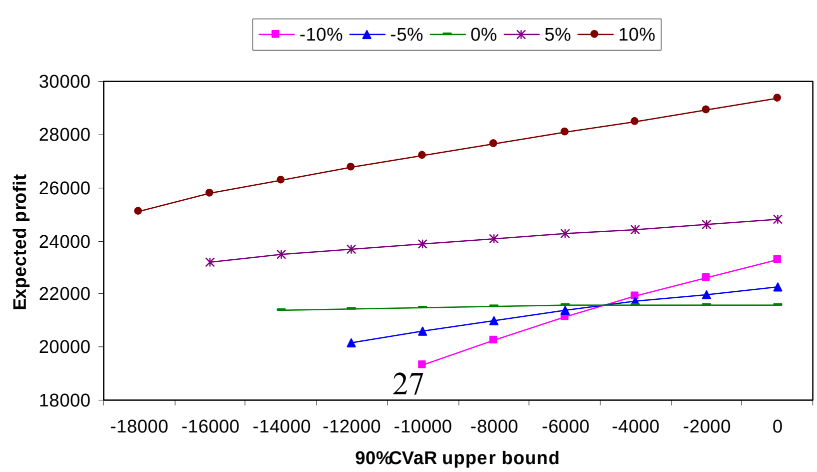


Figure 3. Effect of biased futures prices on efficient frontiers in the three ENSO phases

Figure 3 shows the mean-CVaR efficient frontiers associated with various biased futures prices for different ENSO phases. With the efficient frontiers, the farmer makes the optimal decision based on the trade-off between expected profit and downside risk while staying within his/her downside risk tolerance. The three graphs show that the Neutral phase has the highest expected profit and lowest feasible CVaR upper bound. In contrast, La Niña phase has the lowest expected profit and highest feasible CVaR upper bound. The pattern of the efficient frontiers in the three graphs is the same. Higher positive bias of the futures price leads to higher expected profit. However, the higher negative bias of futures price provides a higher expected profit under a looser CVaR constraint and a lower expected profit under a stricter CVaR constraint.

5. Conclusion

This article proposed a mean-CVaR model for investigating the optimal crop planting schedule and hedging strategy when crop insurance and/or futures contracts are available for hedging yield and price risks. Due to the linear property of CVaR, the optimal planting and hedging problem can be formulated as a mixed 0-1 linear programming problem that can be efficiently solved by many commercial solvers such as CPLEX. The mean-CVaR model is powerful in the sense that it inherits the advantage of the return versus risk framework (Markowitz, 1952) and further utilizes CVaR as a (downside) risk measure that can cope with general loss distributions. Compared to using utility functions for modeling risk aversion, the mean-CVaR model provides a more intuitive way to define risk. In addition, a problem with nonlinear side constraints can also be formulated linearly under the mean-CVaR framework, which can be solved more efficiently compared to the nonlinear formulation from the utility function framework.

A case study was conducted using data of a representative cotton producer in Jackson County, Florida, to examine the optimal crop planting schedule and risk hedging strategy under the three ENSO phases. The available hedging instruments for cotton included futures contracts and three types of crop insurance policies: APH, CRC, and CAT. When crop insurances are the only available hedging instruments, the 65%CRC or 70%CRC was the optimal insurance coverage as the CVaR constraint becomes strict. Furthermore, the optimal hedging strategy was examined when crop insurance policies and futures contracts are available. When futures prices are unbiased or positively biased, no crop insurance policy is desirable and the optimal hedging strategy only includes futures contracts. However, when the futures prices are negatively biased, the optimal hedging strategy depends on the ENSO phases. Otherwise, the optimal hedging strategy includes only futures contract. In La Niña phase, the optimal hedging strategy contains only futures contract for all CVaR upper bound values and all biases of futures prices in a significant range. The optimal futures hedge ratio increases with the CVaR upper bound when the insurance strategy is unchanged. For a fixed CVaR upper bound, the optimal hedge ratio increases when the positive bias of futures price increases. However, when the futures price is negatively biased, the optimal hedge ratio depends on the value of CVaR upper bound.

The case study provides some insight into how planting schedule, insurance and futures hedging may account for the downside risk of a loss distribution. The small sample size for the El Niño and La Niña phases may limit the case study results. In addition, the cost of futures contract was assumed to be the commission plus an average interest foregone for margin deposit. The risk of daily settlement, which may require a large amount of cash for margin account, was not considered. This may reduce the value of futures hedging for risk-averse farmers.

References

Artzner, P., F. Delbaen, J.-M. Eber, and D. Heath. 1999. "Coherent measures of risk". *Mathematical Finance*, 9, 203–228.

Cabrera, V.E., C. Fraisse, D. Letson, G. Podestá, and J. Novak. 2006. Impact of climate information on reducing farm risk by optimizing crop insurance strategy. Transactions of the American Society of Agricultural and Biological Engineers (ASABE) 49: 1223-1233.

Cabrera, V.E., D. Letson, and G. Podestá. 2007. The Value of the Climate Information when Farm Programs Matter. Agricultural Systems 93: 25-42.

Cane, M.A., G. Eshel, and R. Buckland. 1994. "Forecasting Zimbabwean maize yield using eastern equatorial Pacific sea surface temperature." *Nature* 370, 204-205.

Chambers, R.G. and J. Quiggin. 2002. "Optimal producer behavior in the presence of area-yield crop insurance." *American Journal of Agricultural Economics*. 84, 320-334.

Coble, K. H., J.C. Miller, and M. Zuniga. 2004. "The joint effect of government crop insurance and loan programmes on the demand for futures hedging." *European Review of Agricultural Economics*. 31, 309-330

Hansen, J.W., A.W. Hodges, and J.W. Jones. 1998. "ENSO influences on agriculture in the Southeastern US." *Journal of Climate* 11, 404-411.

Hansen, J.W. 2002. "Realizing the potential benefits of climate prediction to agriculture: issues, approaches, challenges." *Agricultural Systems* 74, 309-330.

Jones, J.W., J.W. Hansen, F.S. Royce, and C.D. Messina. 2000. "Potential benefits of climate forecast to agriculture." *Agriculture, ecosystems & environment.* 82, 169-184.

Jones, J.W., G. Hoogenboom, C.H. Porter, K.J. Boote, W.D. Batchelor, L.A. Hunt, P.W. Wilkens, U. Singh, A.J. Gijsman, J.T. Ritchie. 2003. "The DSSAT cropping system model." *European Journal of Agronomy.* 18, 235-65.

Jorion, P. 2000. *Value at Risk: The New Benchmark for Managing Financial Risk*, 2nd. ed. New York: McGraw-Hill Book Co.

Kruksal, W.H. 1958. "Ordinal measures of association." *Journal of the American Statistical Association*. 53, 814-861.

Mahul O. 2003. "Hedging Price Risk in the Presence of Crop Yield and Revenue Insurance." *European Review of Agricultural Economics*, 30, 217-239.

Markowitz, H.M. 1952. "Portfolio Selection." *Journal of Finance*, 7, 77–91.

McKinnon, R. I. 1967. "Futures Market, buffer stocks, income stability for primary producers." *Journal of Political Economy.* 75, 844-861.

Messina, C.R., J.W. Jones, C.W. Fraisse. 2005. "Development of cotton CROPGRO crop model." *Southeast Climate Consortium Staff Paper Series 05-05*. Gainesville, FL.

Nemhauser, G.L. and L.A. Wolsey. 1999. *Integer and Combinatorial Optimization*, New York: Wiley Book Co.

Poitras, G. 1993. "Hedging and Crop Insurance." *The Journal of Futures Markets.* 13, 373-389.

Rockafellar R.T. and S. Uryasev. 2002. "Conditional value-at-risk for general loss distribution." *Journal of Banking and Finance.* 26, 1443-1471.

——. 2000. "Optimization of Conditional Value-At-Risk." *The Journal of Risk*, 2, 21-41.

Sklar A. 1959. "Fonctions de répartition à *n* dimensions et leurs marges." *Publications de l'Institut de Statistique de L'Université de Paris.* 8, 229-231.

Szegö G. 2002. "Measures of Risk." *Journal of Banking and Finance*, 26, 1253–1272.