

# PREDICTION OF MILKLINE FILL AND TRANSITION FROM STRATIFIED TO SLUG FLOW

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Published in the 1995 Transactions of the ASAE. Vol. 38(3): 975-978.

## **ABSTRACT:**

Equations were developed to predict milkline fill and the transition from stratified to slug flow under normal milking conditions. The equation to predict fill explained 93 percent of the variation in the experimental data for 48 to 98 mm milklines sloped at 0.5 to 2 percent. The prediction equation for the transition from stratified to slug flow accounted for 98 percent of the variation in the experimentally determined transition points.

## **INTRODUCTION**

Equations developed to describe open channel flow have been used to predict fill in milklines (Gates, 1980, Gates et al, 1981, and 1982). These studies used Manning's equation to relate milkline diameter, slope and fill with milk flow. Manning's equation is derived from a force balance on a fluid element between gravity and the friction between the fluid and pipe wall. The momentum transfer between the milk and air at the free surface is neglected. Furthermore, it is usually assumed that the flow is fully developed.

In most milking systems, air flows in the milkline above the milk, generally in the same direction as milk flow. Air enters milklines in two ways during milking. A steady flow of air is introduced through designed air vents in clusters and some types of milk meters, and through leaks at fittings. Air admission through air vents is constant during the milking process and accounts for 8 to 15 L/min (0.3 to 0.5 cfm) per milking unit. Air is also admitted intermittently (transient air) when units are attached to the cow, or when cups slip or fall off.

For a constant milk flow rate, momentum transferred from the steady air flow in the free space above the milk acts to decrease milkline fill under stratified flow conditions. Momentum transferred from intermittent air admission will either further reduce fill or cause slugging in the milkline.

Momentum transfer from air to milk increases as the depth of milk in the pipe increases and the area available for air flow decreases. Air velocity can be an order of magnitude above milk

velocity under normal operating conditions. Analyses by Gates (1980) showed that when the volumetric air to milk flow ratio and pipe fill are high, the effects of momentum transfer are as large or larger than the opposing milk/wall frictional force. These are the conditions encountered near the point of transition from stratified to slug flow.

Milk enters the milkline from individual milking units at intervals of 65 to 120 cm (27 to 48 inches) in milking parlors and up to 240 cm (96 inches) in round-the-barn pipeline systems. The milk enters from the milk hose at high velocity compared with the flow velocity in the milkline and in a direction perpendicular to the flow direction in the milkline. This creates a region of intense mixing at each milk inlet and induces resistance to the flow of milk in the milkline. It may take up to 300 pipe diameters for the flow to fully develop and for the effects of this mixing region to decay.

The milk flow rate in the milkline increases nearer the receiver as more units add milk to the line. Maximum fill is expected to occur just downstream of the milk inlet nearest the receiver, which is a region of undeveloped flow. This is the critical location for the development of slug flow. Stratified flow is characterized by milk flow in the lower portion of the pipe and air flow in the upper portion. Slug flow occurs when milk fills the entire pipe cross section forming "slugs" of varying length. The objectives of this study were to develop equations to predict:

- 1) milkline fill, considering the effects of mixing and airflow and,
- 2) the onset of slugging in milklines under normal operating conditions.

## **METHODS**

An experimental study was performed to determine the point of transition from stratified to slug flow conditions for 48 , 73, and 98 mm ID (2", 3" and 4" nominal) milklines (Reinemann and Mein, 1994). The experimental conditions and measurement points were chosen to replicate a worst-case scenario for the expected maximum fill and slugging conditions. Both steady and transient air flows were admitted to the milkline in this experiment. Tests were performed over a range of steady air to milk flow ratios from 1.5 to 3.0 with most tests performed with a ratio of 2.2:1. This represents the range of steady air to milk flow ratios under peak flow conditions for commercial milking units and fast milking cows (Mein et al, 1993). Milkline fill was measured just upstream from the elbow nearest the receiver (1.5 m downstream of the nearest milk inlet). The transition between stratified and slug flow was determined for both steady air flow and a combination of steady and transient air flows as encountered in milking systems.

## **MILKLINE FILL PREDICTION**

Manning's equation, developed to predict open channel flow, is usually presented in the

$$Q_m = \frac{A_m R_m^{2/3} S^{1/2}}{n}$$

following form:

where:

- $Q_m$  = Volumetric milk flow rate (L/min)
- $A_m$  = Cross sectional area of milk flow ( $m^2$ )
- $R_m$  = Hydraulic radius of milk (m)
- $S$  = Milkline slope (%)

This equation is derived from a force balance on a fluid element flowing in an open channel, neglecting interfacial forces at the milk / air interface. Experimental data for stratified flow conditions for a 73 mm (3") milkline were used to determine the value of  $n$  in (1). The average value of  $n$  for 171 data points was 0.11 and the correlation coefficient ( $R^2$ ) was 0.13. Calculated values of  $n$  are plotted against the air velocity in the free space above the milk in Figure 1.

At low air velocity, the value for  $n$  of 0.16 is considerably higher than values used for fully developed, open channel flow. Gates found a value of 0.09 in a study in which pipe fill was measured far from the point of milk entry, where flow was fully developed. The higher value of  $n$  found here at low air velocity is the effect of increased friction caused by mixing in the region of undeveloped flow near milk inlets.

Increased air velocity reduced the value of  $n$ . The form of the decrease (proportional to the square of the velocity) is as expected for frictional transfer of momentum from air to milk.

The effects of gravity, milk/wall friction and air/milk momentum transfer were included in a force balance on a fluid element to give:

$$S = \frac{f_m V_m^2}{2 g R_m} \frac{f_a \rho_a (V_a V_m)^2}{\rho_m 2 g R_a}$$

- where:  $f_m$  = milk / wall friction factor  
 $V_m$  = milk flow velocity (m/s)  
 $g$  = acceleration due to gravity ( $m/s^2$ )  
 $f_a$  = air/milk friction factor  
 $\rho_a$  = air density ( $kg/m^3$ )  
 $V_a$  = steady air velocity (m/s)  
 $\rho_m$  = milk density ( $kg/m^3$ )  
 $R_a$  = hydraulic radius of air (m)

Neglecting the milk velocity compared with the air velocity in the second term, this equation can be rearranged in the form of Manning's equation as:

$$S = \frac{C_1 Q_m^2}{R_m^{4/3} A_m^2} \frac{C_2 Q_{aSteady}^2}{R_a^{4/3} A_a^2}$$

$Q_{a\text{Steady}}$	=	steady volumetric air flow rate measured at standard conditions (L/min)
$A_a$	=	cross sectional area for air flow ( $\text{m}^2$ )
$C_1$	=	milk friction coefficient
$C_2$	=	air friction coefficient

This equation can be simplified by substituting the following expression for the hydraulic radii

$$R_m = 0.35 D F^{1/2}$$

$$R_a = 0.79 D F$$

for circular pipe cross sections:

where:  $D$  = Inner diameter of pipe (m)

$F$  = Fraction of pipe cross-sectional area filled with milk

The expression for the hydraulic radius of the air is exact. The expression for the hydraulic radius of the liquid phase is within 1% of actual value for fills from 20 to 80 percent.

Substituting (4) and (5) in (3) and rearranging gives:

$$Q_m^2 = C_3 S D^{16/3} F^{8/3} + C_4 \frac{Q_{a\text{Steady}}^2 F^{4/3}}{(1 F)^2}$$

Milk fill from 30 to 70 percent is the range in which the transition to slug flow will occur for the type of air flows encountered in milking systems. Experimental data for 48, 73, and 98 mm inner diameter pipes sloped at 0.5, 1.0 and 2.0 percent with fills from 20 to 70 percent were regressed to determine the coefficients  $C_3$  and  $C_4$  in equation (6) with the following result:

$$C_3 = 2.4 \times 10^{-6}$$

$$C_4 = 1.4 \times 10^{-2}$$

Number of data points ( $n$ ) = 171

Correlation Coefficient ( $R^2$ ) = 0.93

## TRANSITION FROM STRATIFIED TO SLUG FLOW

Flow pattern maps are commonly use to define the boundary between flow regimes in two phase flow. The boundary lines are commonly expressed in terms of superficial liquid and air velocities. Superficial velocity is obtained by dividing the volumetric flow rate of the phase (liquid or gas) by the total pipe cross sectional area. Regression analysis was performed to determine the coefficient and exponents of a transition boundary between stratified and slug flow for milk lines. Data from 48, 73, and 98 mm inner diameter pipes sloped at 0.5, 1.0 and 2.0 percent were used ( $n = 33$ ,  $R^2 = 0.88$ ).

$$V_{Sa} = \frac{0.016 s^{1.03} d^{0.83}}{V_{Sm}^{0.92}}$$

where: d = Internal pipe diameter (mm)  
 $V_{Sa}$  = Superficial air velocity (m/s)  
= Total volumetric air flow rate (steady + transient air) measured at milking system vacuum / total pipe cross-sectional area.  
 $V_{Sm}$  = Superficial milk velocity (m/s)  
= Volumetric milk flow rate / total pipe cross sectional area.

This equation was simplified to the following form without significant reduction in prediction accuracy (n = 33,  $R^2 = 0.86$ ):

$$V_{Sa} = \frac{0.007 s d}{V_{Sm}}$$

An easier to use form of (8) using volumetric milk and volumetric air flow rate at standard conditions was also developed (n = 33,  $R^2 = .98$ ):

$$Q_m = \frac{8.9(10)^6 S d^5}{Q_{aTotal}}$$

where:  $Q_{aTotal}$  = Total (steady + transient) volumetric air flow rate per milking line slope, measured at standard conditions (L/min)

Steady air to milk flow ratios between 1.5:1 and 3:1 were tested experimentally as this covers the range of claw air admission and peak milk flow rates commonly encountered in practice. Over this range of steady air to milk flow, experimentally determined milk flow rate at the flow transition points varied by less than 5%. To simplify the calculations, therefore, a constant steady air to milk ratio of 2.2:1 (e.g. 10 L/min steady air flow per 4.5 L/min milk flow) was used

$$Q_{aTotal} = Q_{aTransient} + 2.2 Q_m$$

to derive  $Q_m$  as follows:

where:  $Q_{aTransient}$  = Transient volumetric air flow rate per milking line slope, measured at standard conditions (L/min)

Table I. Example predicted flow conditions at the transition from stratified to slug flow for a 73 mm milking line with steady air to milk flow ratio of 2.2, using (9) and (10).

Milkline Slope (%)	Steady Air Flow Rate (L/min)	Transient Air Flow Rate (L/min)	Total Air Flow Rate (L/min)	Predicted Milk Flow Rate (L/min)
0.5	101	100	201	46
1.0	158	100	258	72
2.0	239	100	339	109

Equation (10) was then substituted in (9) and solving for the positive root of the quadratic equation for  $Q_m$  (Table I). Experimentally determined transition points are plotted with predicted values using (9) and (10) in Figures 2 - 4.

## SUMMARY AND CONCLUSIONS

The effect of mixing at the point of milk entry produces a significant increase in pipe wall friction and fill compared with fully developed flow. The effects of momentum transfer between air and milk on milkline fill are significant in the range of interest for milking systems.

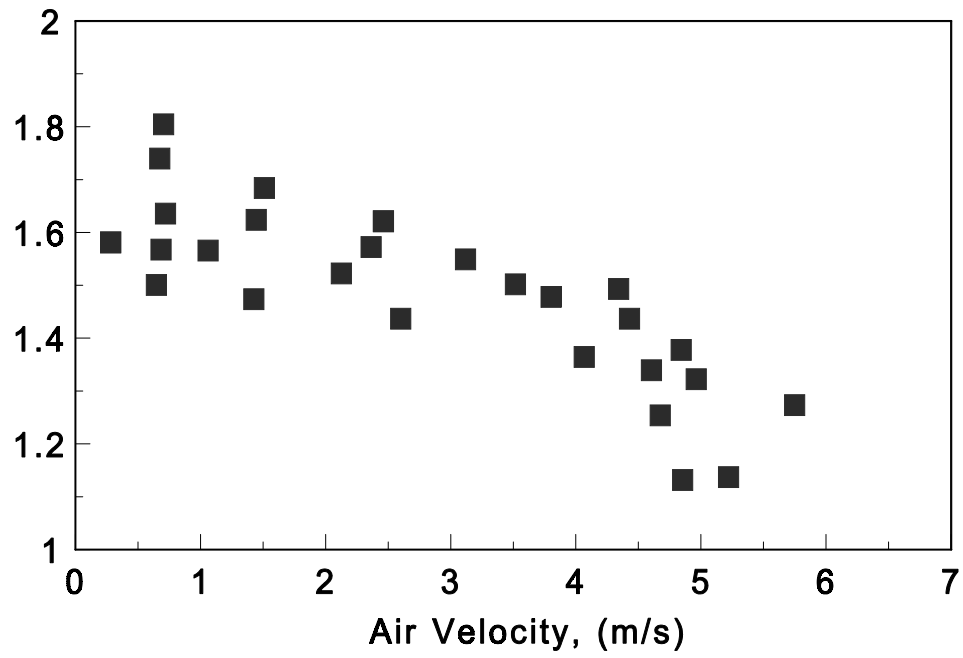
Manning's equation and other

versions of fully developed, open channel flow equations are not accurate predictors of milkline flow dynamics because of these two effects. A new equation (6) was developed to account for these two effects. This equation explained 93 percent of the variation in the experimental data for 48 to 98 mm milklines sloped at 0.5 to 2 percent. An equation was also developed to predict the transition from stratified to slug flow (9). This equation accounted for 98 percent of the variation in the experimentally determined transition points.

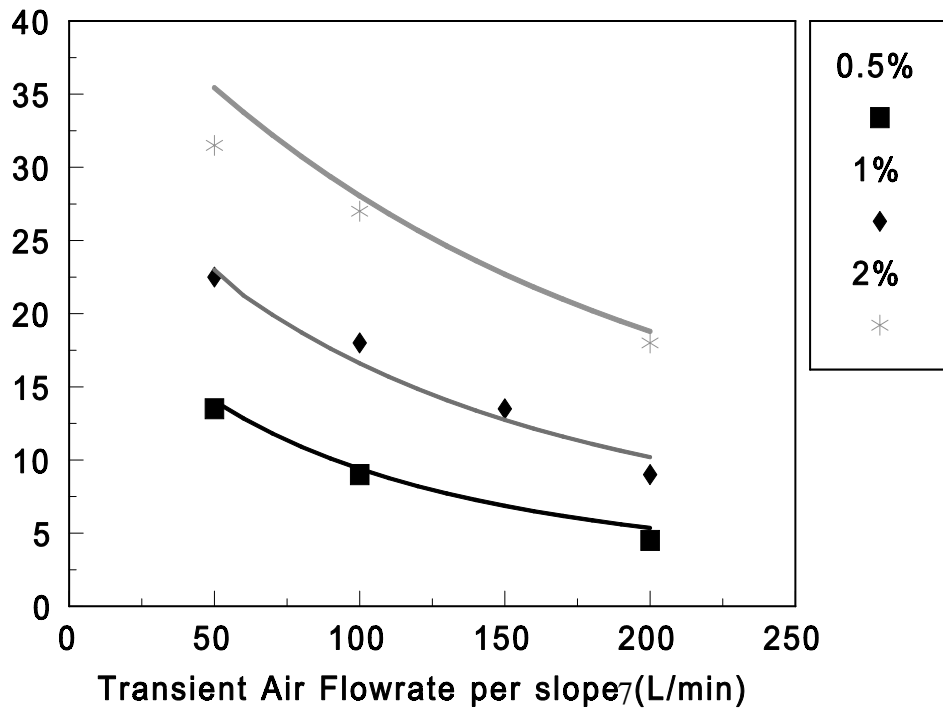
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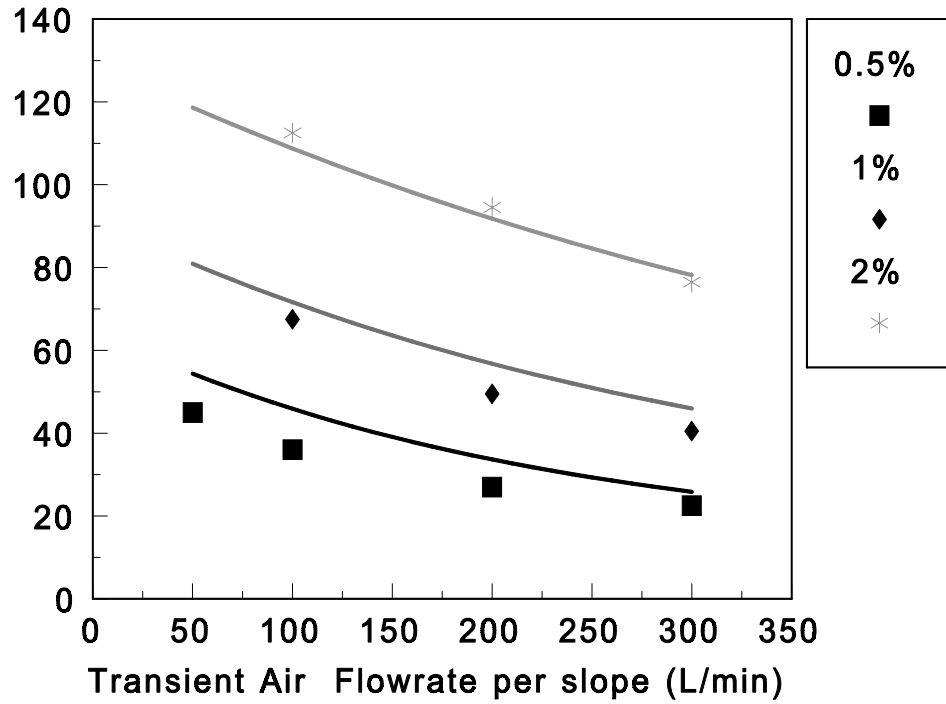
Manning's n  
(Times 1E-2 )



Milk Flowrate per slope (L/min)



Milk Flowrate per slope (L/min)



Milk Flowrate per slope (L/min)

